UConn ECE4211 HW#3 01/31/2017, Due 02/07/17, F. Jain, NAME SOLUTIONS ①

HW#2 Q. 11-Q12 to be solved as of part of this assignment. HW 3 has 4 questions.

HW2-Q11. Find the coefficient A and B for a p-n junction diode shown in Fig. 4 with a finite neutral n-region l_n.

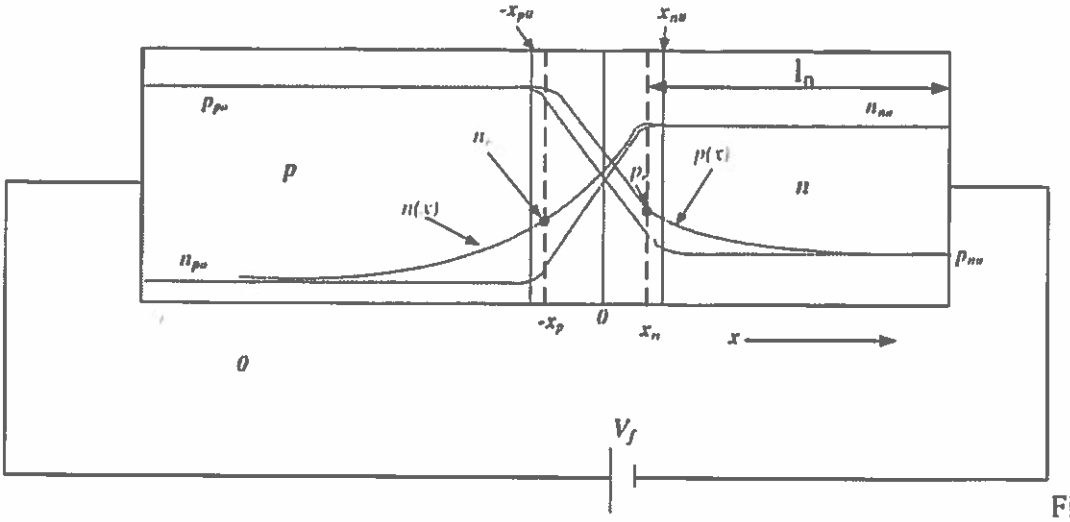


Fig. 4.HW2

Carrier distribution equation (53A) $\delta p(x) = A e^{\frac{-(x-x_n)}{L_p}} + B e^{\frac{(x-x_n)}{L_p}}$ for $\infty > x > x_n$

Boundary conditions (BCs) are: p_n at $x = x_n$ is $p_e = p_{no} e^{\frac{qV_f}{kT}}$

OR $p_n(x = l_n) = p_{no}$

(BC#1) $\delta p(x = x_n) = \Delta p = p_e - p_{no} = p_{no}(e^{\frac{qV_f}{kT}} - 1)$, and BC#2 $\delta p(x = l_n) = 0$.

Here, ln is the length of n-region. The n-region is finite (not ∞).

Find A, B. using BC#1 gives $A + B = p_{no}(e^{\frac{qV_f}{kT}} - 1)$, and BC#2 gives $A e^{\frac{-(l_n - x_n)}{L_p}} + B e^{\frac{(l_n - x_n)}{L_p}} = 0$

$A = -B \dfrac{e^{\frac{(l_n - x_n)}{L_p}}}{e^{-\frac{(l_n - x_n)}{L_p}}} = -B e^{\frac{2(l_n - x_n)}{L_p}}$

$-B e^{\frac{2(l_n - x_n)}{L_p}} + B = p_{no}(e^{\frac{qV_f}{kT}} - 1)$

$B = p_{no}(e^{qV_f/kT} - 1) / \left(1 - e^{\frac{2(l_n - x_n)}{L_p}}\right)$

$A = -e^{\frac{2(l_n - x_n)}{L_p}} \left[p_{no}(e^{\frac{qV_f}{kT}} - 1) / \left(1 - e^{\frac{2(l_n - x_n)}{L_p}}\right) \right]$

HW2-Q12 (a) Write expression of junction capacitance C_J at:

Equilibrium (W_o) C_J — P.106 Eq 40B

Forward bias (W_f) C_J — L11 A.

Reverse bias W_r. C_J — L11 B.

$C_J = \frac{\epsilon_s \epsilon_o A}{W}$

$W_o = \left[\frac{V_{bi} \times 2\epsilon_r\epsilon_o (N_A + N_D)}{q N_D N_A}\right]^{1/2}$

$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$

we have used V_{bi}

$W_f = \left[\frac{2(V_{bi} - V_f)\epsilon_r\epsilon_o (N_A + N_D)}{q N_A N_D}\right]^{1/2}$

$W_r = \left[\frac{2(V_{bi} + V_r)\epsilon_r\epsilon_o (N_A + N_D)}{q N_A N_D}\right]^{1/2}$

(b) Write expressions for diffusion capacitance C_{diff} under forward bias.

Method I — P.119 (n-p)

$Q_n = A q n_{po} L_n (e^{\frac{qV_f}{kT}} - 1)$

Method II

$Q_{p-n} = A q p_{no} L_p (e^{\frac{qV_f}{kT}} - 1)$

$C_{DIFF} = \frac{dQ_n}{dV_f}$ or dQ_p/dV_f

Eq 86 —

$Y(j\omega) = \left[(q p_{no}/L_p)(1 + j\omega\tau_p)^{1/2} + q D_n n_{po}(1 + j\omega\tau_n)^{1/2}\right] \frac{1}{(1 + \frac{q}{kT})}$

Eq. 91 — C_{DIFF} = Imaginary part of $Y(j\omega) * \frac{1}{j\omega}$

HW3-Q.1 Fig. 1 shows a p-n junction under equilibrium. The device doping parameters are as shown in Fig. 3. Other parameters are as follows:

Given n_i in Si at room temperature (T=300K) $=1.5\times10^{10}$ cm^{-3}; the product of hole concentration p and electron concentration n outside the junction is constant $pn = (n_i)^2$ Assume all donors and acceptors are ionized at 300K.

n-side: Donor concentration $N_D=10^{15}$ cm^{-3}, minority hole lifetime $\tau_p=2\times10^{-6}$ sec.

Minority hole diffusion coefficient $D_p=12.5$ cm^2/sec.

p-side: Acceptor concentration $N_A=10^{18}$ cm^{-3}, $\tau_n=10^{-5}$ sec. $D_n=40$ cm^2/sec.

Effective mass: electrons $m_e=m_n=0.26m_o$, holes $m_h=m_p=0.64\ m_o$,

Junction area $A=10^{-3}$ cm^{-2}, n_i (300K)$=1.5\times10^{10}$ cm^{-3}. ε_r(Si)=11.8, $\varepsilon_0=8.85\times10^{-14}$ F/cm, $\varepsilon_s=\varepsilon_r\varepsilon_0$.

Assume all donors and acceptors are ionized at T=300 K. E_g = 1.1eV, kT/q = 0.0259V

(a) Find junction width W_o and its components x_{no} and x_{po}. HINT: Eqs. 33-35 page 103-104.

(b) Draw the junction boundaries for a forward biased p-n junction. For this you need to find the new junction width. Proportionately decrease the values and obtain new x_n and x_p values. HINT: Use junction width $W=W_f$ under forward bias page 106 Eq. 41A; $V=V_f=0.5$V. New Eq. $qAN_Dx_n = qAN_A x_p$

(c) Find hole concentration p_e at the n- region boundary x_n and electron concentration n_e at $-x_p$ under a forward bias of 0.7V.HINT: Use Eqs. 55 and 61 (pages 109-110). Electrons on p-side $n_p(x=-\infty)=n_{po}$ and $n_e=n_p(-x_p)$.

(d) Write the boundary conditions for carrier concentration at the junction boundaries for holes and electrons and at infinite distance away from the junction. We are assuming both n and p regions are much larger than the diffusion lengths L_p (for holes diffusing in n-region) and L_n for electrons diffusing in the p-region.

(e) Find and draw the carrier concentration profile $p_n(x)$ and $n_p(x)$ as a function of x using the diffusion coefficient information..

Q.1 (e) Solution. $V_f = 0.7V$

$P_e = 2.25$

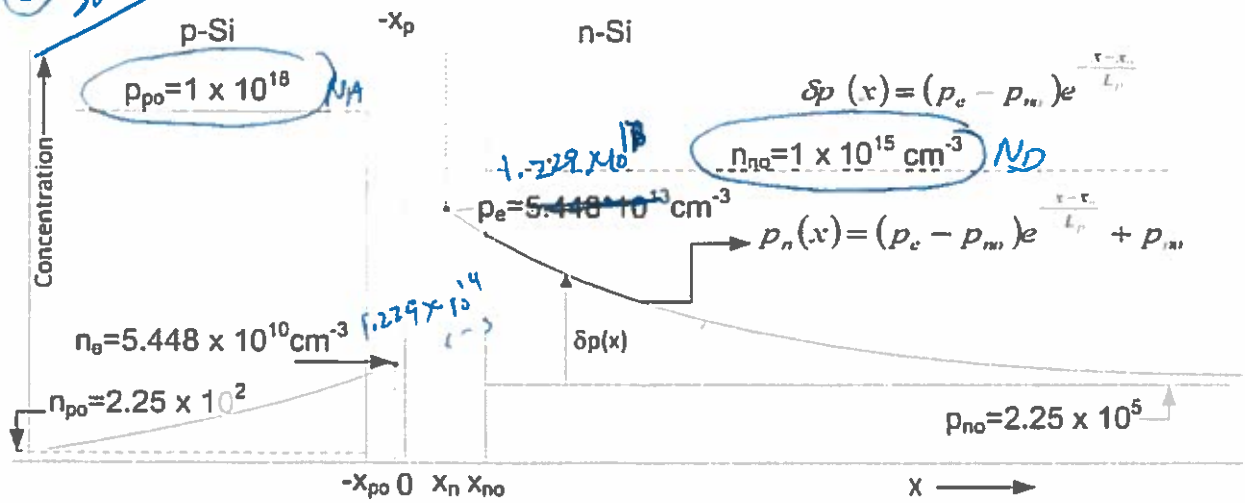


Figure 2. Electron and hole concentration distribution in a forward biased diode.

Extra ~~Q.1(e)~~ What are the boundary conditions for a reverse biased junction with V_r=-0.2V.

Hole concentration $p_e = p(x_{nr}) = p_{no}e^{(-qV_r/kT)} = 2.25\times10^5 * e^{(-0.2/0.0259)} = 99.66cm^{-3}$.

And, electron concentration $n_e = n_{po}e^{(-qV_r/kT)} = 2.25\times10^2 * e^{(-0.2/0.0259)} = 0.09966cm^{-3}$.

If V_r is increased to -2V or so, the edge concentration will approach zero.

0 $\Leftarrow P_e = 2.25\times10^5 \times e^{(-2/.0259)}$ at $V_r = -2V$

Q.1 (f) Draw the carrier concentration distribution after finding the junction width.

Reverse biased junction width W_r at V_r=-0.2V is

Following the procedure to find W_r, we get

$W_r = W(eqb) * [(V_{bi} + V_r)/V_{bi}]^{1/2}$

$W_r = 0.9928[(0.754+0.2)/0.754]^{1/2} = 1.1167\mu m$

$P_e = 6.54 \times 10^{-79} \simeq 0$

$n_e \simeq 6.54 \times 10^{-26} \simeq 0$

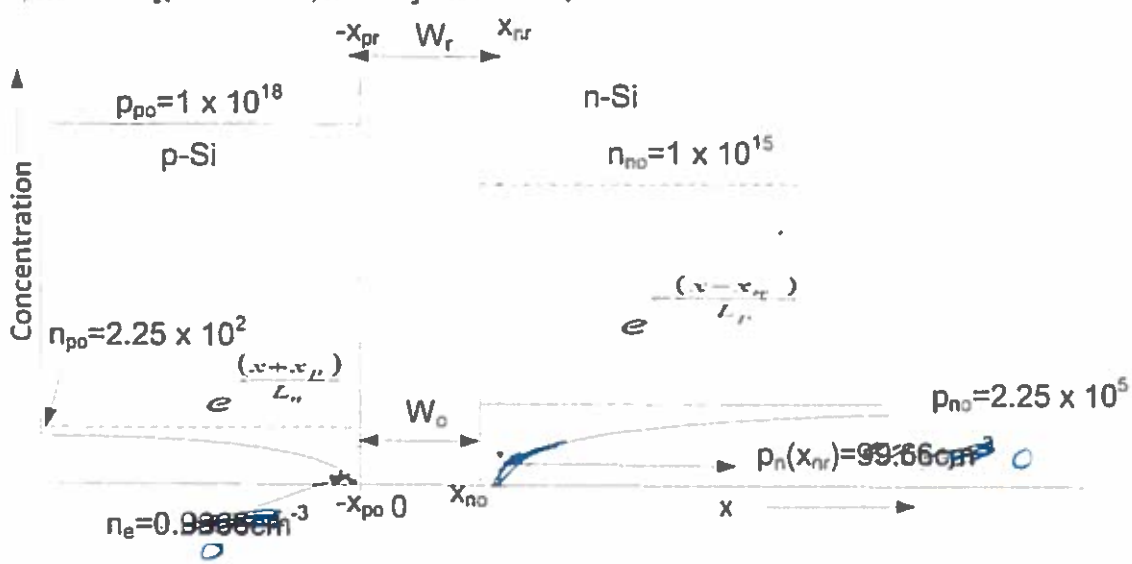


Fig. 3. Carrier distribution in a slightly reverse biased diode V_r = -0.2V.

Q. 1(g) Hole Current

$$I_p(x_n) = \frac{qAD_p p_{no}}{L_p}\left(e^{\frac{qV_f}{kT}} - 1\right) \cong \frac{1.6\times10^{-19}\times10^{-3}\times12.5\times2.25\times10^{5}}{5\times10^{-3}}\left(e^{\frac{qV_f}{kT}} - 1\right)$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{12.5\times2\times10^{-6}} = 5\times10^{-3}cm, \text{ and } e^{\frac{qV_f}{kT}} - 1 = e^{\frac{0.5}{0.0259}} - 1 = 2.42\times10^{8} - 1 \approx 2.42\times10^{8}$$

$$I_p = \frac{1.6\times10^{-19}\times10^{-3}\times12.5\times2.25\times10^{5}\times2.42\times10^{8}}{5\times10^{-3}} = 2.179\times10^{-5}\,Amp$$

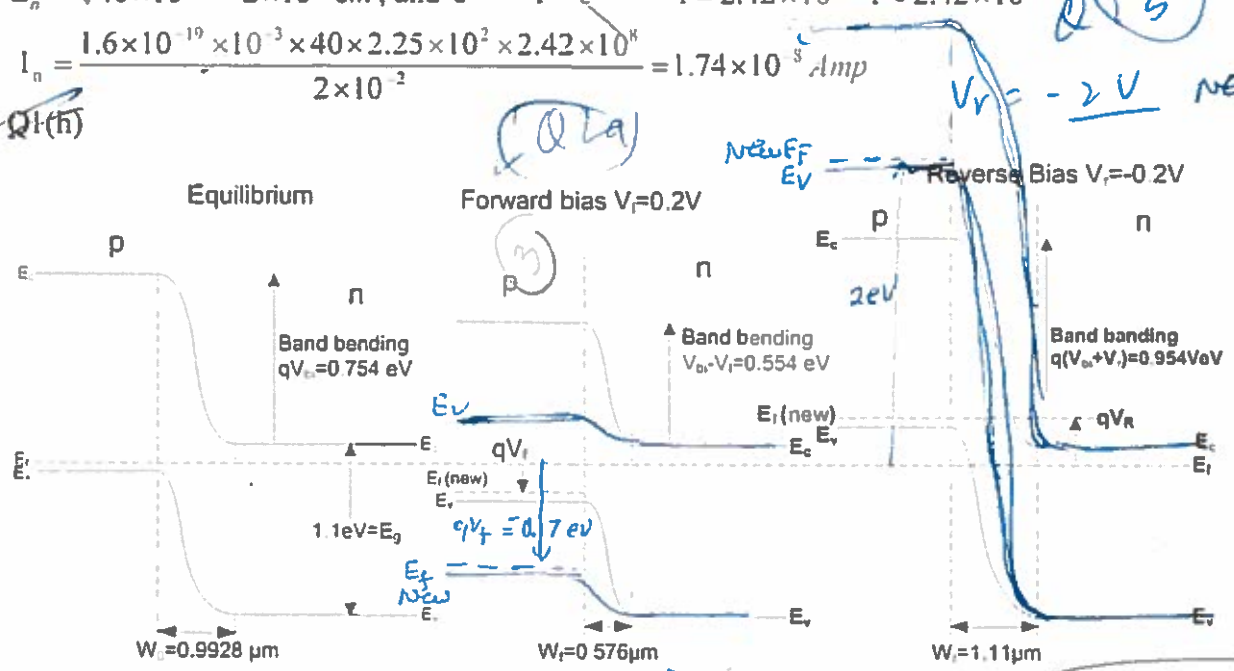
Electron current

$$I_n(-x_p) = \frac{qAD_n n_{po}}{L_n}\left(e^{\frac{qV_f}{kT}} - 1\right) \cong \frac{1.6\times10^{-19}\times10^{-3}\times40\times2.25\times10^{2}}{2\times10^{-2}}\left(e^{\frac{qV_f}{kT}} - 1\right)$$

$$L_n = \sqrt{40\times10^{-5}} = 2\times10^{-2}cm, \text{ and } e^{\frac{qV_f}{kT}} - 1 = e^{\frac{0.5}{0.0259}} - 1 = 2.42\times10^{8} - 1 \approx 2.42\times10^{8}$$

$$I_n = \frac{1.6\times10^{-19}\times10^{-3}\times40\times2.25\times10^{2}\times2.42\times10^{8}}{2\times10^{-2}} = 1.74\times10^{-8}\,Amp$$

Q1(h)



Q1(i)

$$V_{Breakdown} = \frac{E_m W}{2} = \frac{\varepsilon_r\varepsilon_o\varepsilon_m^2}{2q}(N_D)^{-1} = \frac{\varepsilon_r\varepsilon_o\varepsilon_m^2}{2q}\frac{1}{N_D}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{100\times10^{-8}} = 10^{-3}cm \text{ and } N_B \equiv N_D = 10^{15}cm^{-3}$$

Fig. 26 (Ref. S. M. Sze text shows V_{br}=300V) shows the breakdown voltages for various doping levels.

Page 193 Notes

Extra: Solution to Q.1(f) for $N_A = 10^{20}\,cm^{-3}$, $N_D = 10^{17}\,cm^{-3}$

Page 3 of HW 3

From HW2 first question

HW2-Q11. Draw the carrier distribution plots under (a) forward bias V_f=0.7V. **First do HW3-Q1.**

$KT = 8.65\times10^{-5} * 300^\circ K = 0.0259\,eV$

$\frac{KT}{q} = \frac{0.0259\,eV}{q} = 0.0259\,V$

$p_e = p_{no}\, e^{\frac{qV_f}{KT}} = 2.25\times10^{3}\, e^{\frac{0.7}{0.0259}} = 1.229\times10^{15}\,cm^{-3}$

$n_e = n_{po}\, e^{qV_f/KT} = 2.25\times e^{\frac{0.7}{0.0259}} = 1.229\times10^{12}\,cm^{-3}$

(b) Reverse bias of 2V for the p-n junction given below in Fig. 5. **First do HW3-Q1**

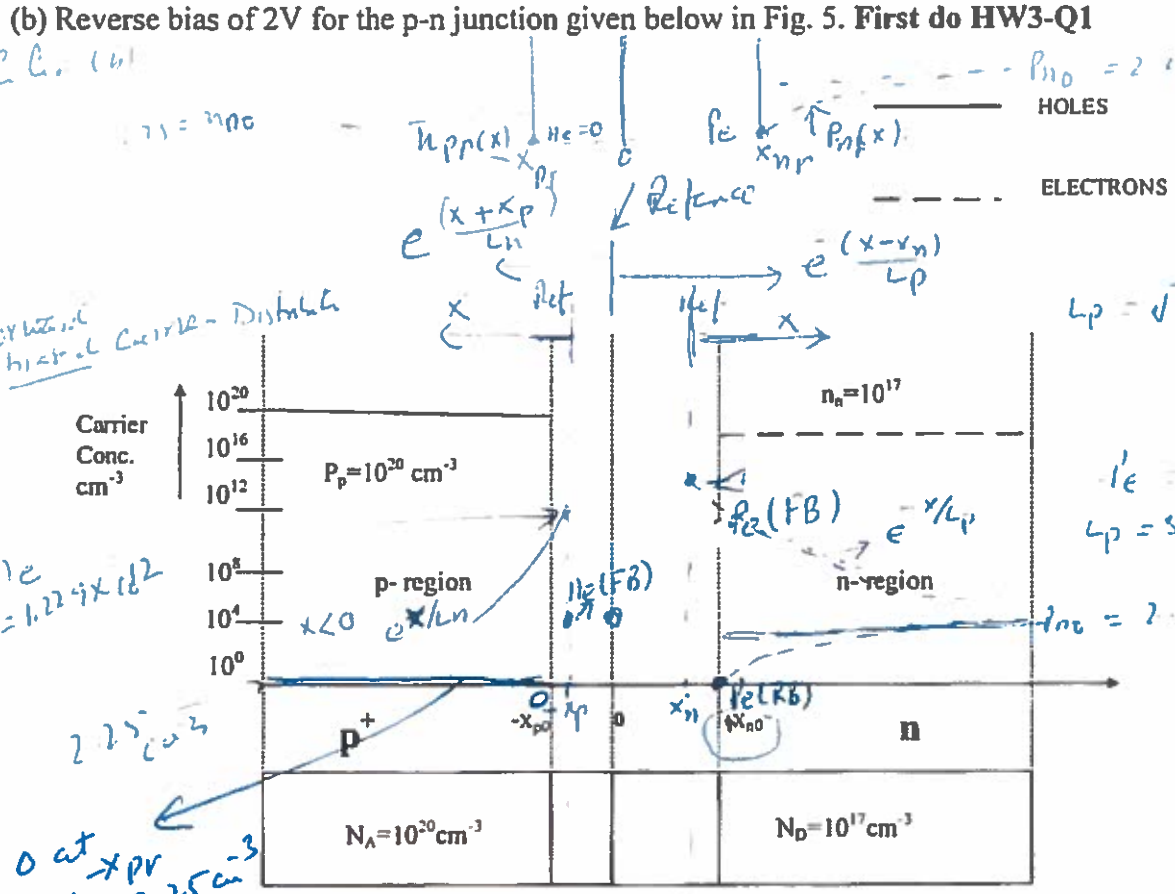
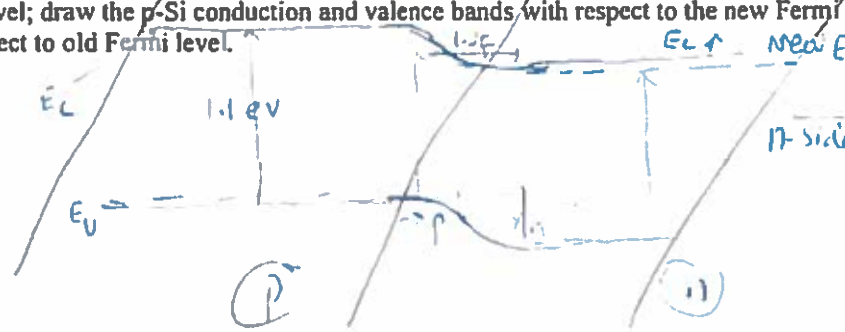


Fig. 5 p-n junction.

HW2-Q12. (a) Draw the energy band diagram under forward bias of 0.7V for the device of Fig. 5.

HINT: Draw the Fermi level as a horizontal line under equilibrium; draw two vertical lines representing the junction with width W_f; draw a new Fermi level for p-side which represents forward bias that is p-side level is qV_f below the n-side Fermi level; draw the p-Si conduction and valence bands with respect to the new Fermi level; draw the n-Si CB and VB with respect to old Fermi level.



n-region is −0.7V with respect to p-region.

n-side is 9 + −0.7 = +0.7eV higher than p-side

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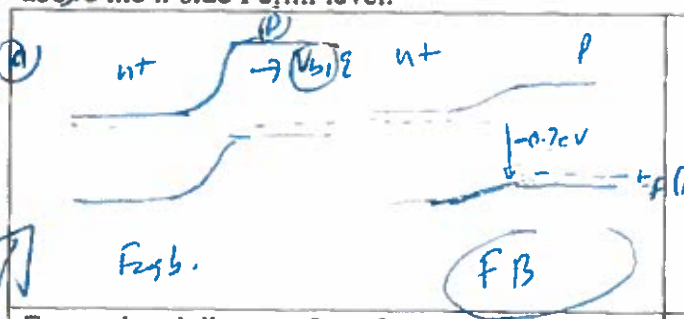
W_f; draw a new Fermi level for p-side which represents forward bias that is p-side level is qV_f below the n-side Fermi level; draw the p-Si conduction and valence bands with respect to the new Fermi level; draw the n-Si CB and VB with respect to old Fermi level.

Q2(b) Draw the energy band diagram for the device of Fig. 2 under reverse bias of 2.0V?

HINT: Draw a new Fermi level for p-side which represents reverse bias that is p-side level is qV_r above the n-side Fermi level.

Q.3(a)



Energy band diagram for a forward biased diode

See. P193

$V_r = -2V$

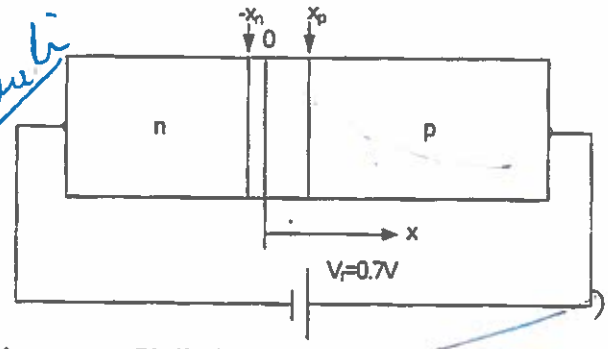
NOT 0.2V

Energy band diagram for a reverse biased diode.

on page 193

Q 3 Figure 3 shows an n^+-p Si diode.

P114 has n-p junction



$p_{no} = \frac{n_i^2}{N_p} = \frac{n_i^2}{10^{20}} = \frac{(1.5\times10^{10})^2}{10^{20}} = 2.25\,cm^{-3}$

Fig. 3 An abrupt n-p Si diode under forward bias.

n^+-side: Donor concentration $N_D=10^{20}$ cm^{-3}, minority hole lifetime $\tau_p=2\times10^{-6}$ sec.
Minority hole diffusion coefficient $D_p=12.5$ cm^2/sec.
p-side: Acceptor concentration $N_A=10^{16}$ cm^{-3}, $\tau_n=10^{-5}$ sec. $D_n=40$ cm^2/sec.
Effective mass: electrons $m_e=m_n=0.26m_o$, holes $m_h=m_p=0.64\,m_o$, $m_o=9.1\times10^{-31}$ kg
Junction area $A=10^{-3}$ cm^{-2}, n_i (300K) $=1.5\times10^{10}$ cm^{-3}. ε_r(Si)=11.8, $\varepsilon_0=8.85\times10^{-14}$ F/cm; $\varepsilon_s=\varepsilon_r\varepsilon_0$.
Assume all donors and acceptors are ionized at T=300 K. E_g = 1.1eV,
Boltzmann Constant k= 8.65×10^{-5} eV/K.

$n_{po} = \frac{n_i^2}{N_A = 10^{16}} = 2.25\times10^{4}\,cm^{-3}$

Q.3

(a) Draw the energy band diagram under equilibrium, forward biasing V_f= 0.7V, and reverse biasing of V_r =-2 V.

(b) Find the reverse situation current I_s.

$I_s = -\left(\frac{qA\,n_{po}D_n}{L_n} + \frac{qA\,p_{no}D_p}{L_p}\right)$ n-side

$I_s = -1.6\times10^{-19}\times10^{-3}\left(\frac{2.25\times10^{4}\times40}{2\times10^{-2}} + \frac{2.25\times12.5}{2.5\times10^{-2}}\right)$ p side

Q.2

(c) Plot current and carrier distribution in the entire diode at V_f= 0.7V. For x axis, plot carrier and current values at x=0 (that is at junction boundary) and x/L_n, $x/2L_n$ and $x/3L_n$ on p-side and $x=-x/L_p$ and $x/2L_p$ and $x/3L_p$ on the n-side.

$p_{no}e^{0.7/0.0259} = p_n$ $\quad n_e = n_{pc}e^{0.7/0.0259}$ $\quad n_{po}$

$2.25 = p_{no}$

$L_p = \sqrt{D_p\tau_p} = 5\times10^{-3}\,cm$

$L_n = \sqrt{D_n\tau_n} = 2\times10^{-2}\,cm$

Done

(d) What happens to the energy when electrons recombine in the p-region?

Photons are emitted along with Phonons emitted / absorbed

BONUS: Q.4(a) Level of doping in a diode that determines avalanche or Zener breakdown. (b) How do you calculate the breakdown voltage (see page 152)? (c) Draw the energy band diagram for a Zener diode. HINT: Zener diode has junction width is about 100Å or smaller.

The second term is much smaller than first

$I_s \simeq -1.6\times10^{-22}\left(\frac{2.25\times10^{4}\times40}{2\times10^{-2}}\right)$

$I_s = 1.6\times10^{-21}\times45\times10^{6} = 7.2\times10^{-15}$ Amp

P13 Zener

4

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Q.4(a) P130 Fig B+Fig.C
When level of p+n doping gives W_n n < 100 Å, we get Zener
Q4(b) Electric field in junction
Q4(c) See Fig 11 p 177