

UConn ECE 4211,
Semiconductor Devices and Nanostructures
Lecture Week 1 January 17, 2017

- **Device Operation:** One of the objectives of this course is to understand operation of carrier transport in semiconductor p-n junction diodes (which form the foundation of light-emitting diodes, lasers, photodetectors and solar cells) and FET-based devices and circuits are used in ICs, memories, and imaging devices such as CMOS digital and CCD cameras.
- **Device Design:** Our next objective is to be able to design a device to specifications that meet the need of a system or larger circuit.
- In order to integrate a device in a system, we need to relate its circuit model to the physical parameters and fabrication methods.
- **Nanostructures:** Incorporate nanostructures (quantum wells, and quantum dots) to enhance performance of FETs, lasers or solar cells, etc.

Resistors

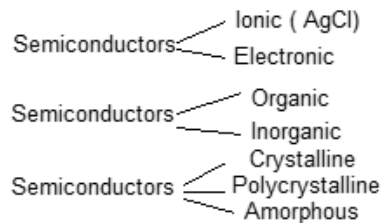
Unlike metals (or good conductors), the resistivity of semiconductors is highly dependent on temperature and level of impurities (doping concentration). A highly purified semiconductor is termed as intrinsic, while semiconductors consisting of impurities (donors and acceptors) are called extrinsic. The carrier concentration (electron n and hole p) depends on doping of impurities and temperature. The product of electron and hole concentration at equilibrium is constant (this is explained later in detail). Once we know one carrier concentration, we know the concentration of the other.

Semiconductors with majority carriers of electrons ($n \gg p$) are referred to as n-type; when the majority carriers are holes ($p \gg n$), they are called p-type. The conductivity is expressed as Eq. 116 as a function of electron and hole concentrations.

N-type: $\sigma_n = (q n_n \mu_n + q p_n \mu_p)$, Eq. 116A, p-type $\sigma_p = (q p_p \mu_p + q n_p \mu_n)$ (116B)

Here, q is the magnitude of electron charge = 1.6×10^{-19} coulomb, n the electron concentration, p the hole concentration, and μ_n and μ_p are the electron and hole mobility, respectively.

Semiconductors are classified in various ways including:



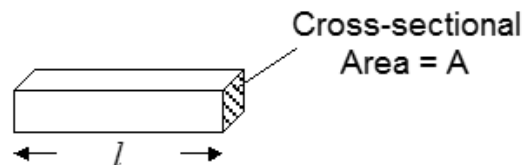
Resistance and Resistivity: The resistance is defined in terms of resistivity and dimensions.

$$R = \rho \frac{l}{A} \quad (122)$$

ρ = Resistivity (unit Ohm-cm (Ω -cm) or Ohm-meter)

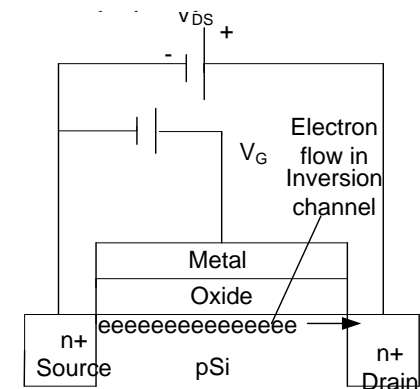
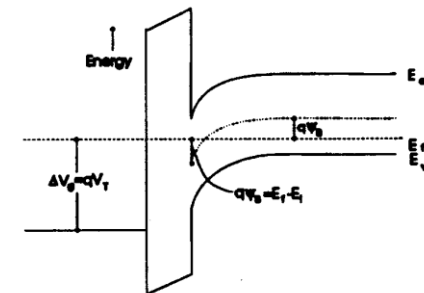
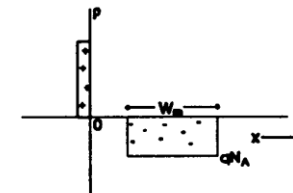
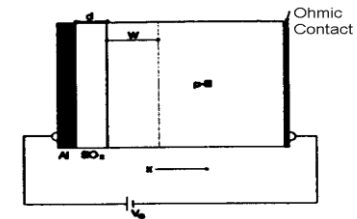
l = length (in cm or meter)

A = Area of Cross-Section



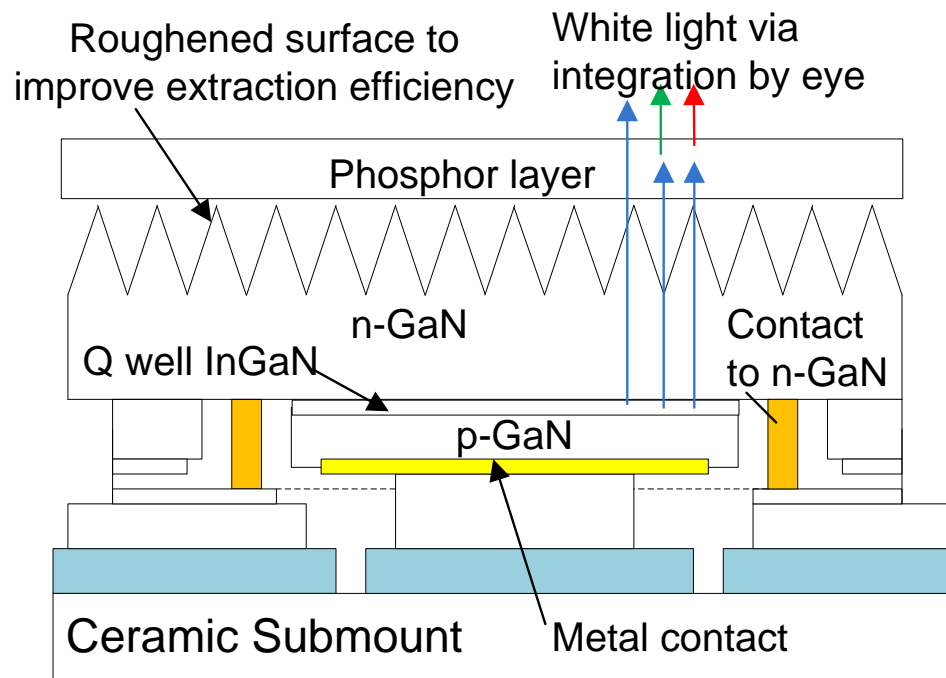
Conductivity $\sigma = 1/\rho$ (unit Mhos per cm)

Capacitors



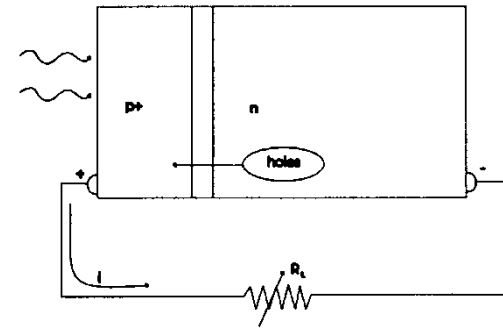
$$I_D = \left(\frac{W}{L} \right) C_o \mu_n \left[(V_G - V_T) V_D - \frac{V_D^2}{2} \right] \quad 2$$

LEDs are used in flash lights, traffic lights, white light sources for solid state lighting.

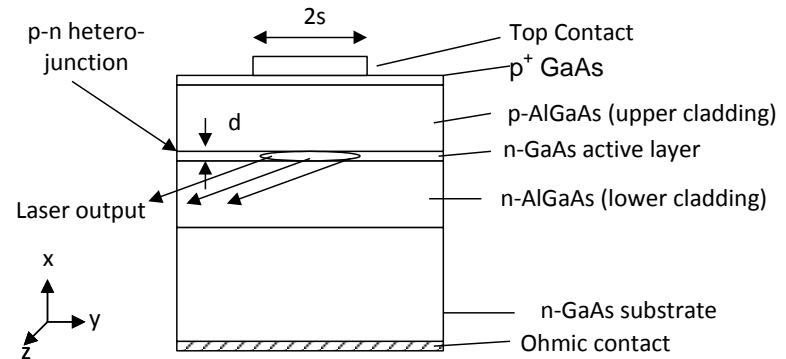


Page 316. Fig. 33 shows a blue LED with green and red phosphor layer to produce white light using flip chip p-GaN fully metalized design. 2009.7 (August 2009) Philips Lumileds Rebel warm white light lamp [4].

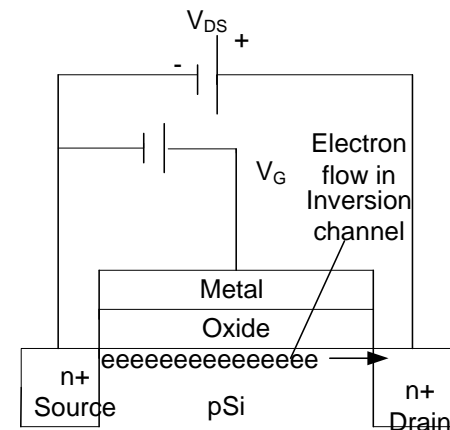
Solar cells that you see mounted on roof tops are p-n junctions.



Laser emission is coherent or stimulated.



FETs are seldom used as stand alone devices. They are used as in logic circuits or analog circuits, analog-to-digital converters (ADCs), static random access memories, nonvolatile memories etc.



Wafer specifications

Semiconductors come in wafers or are deposited as thin films

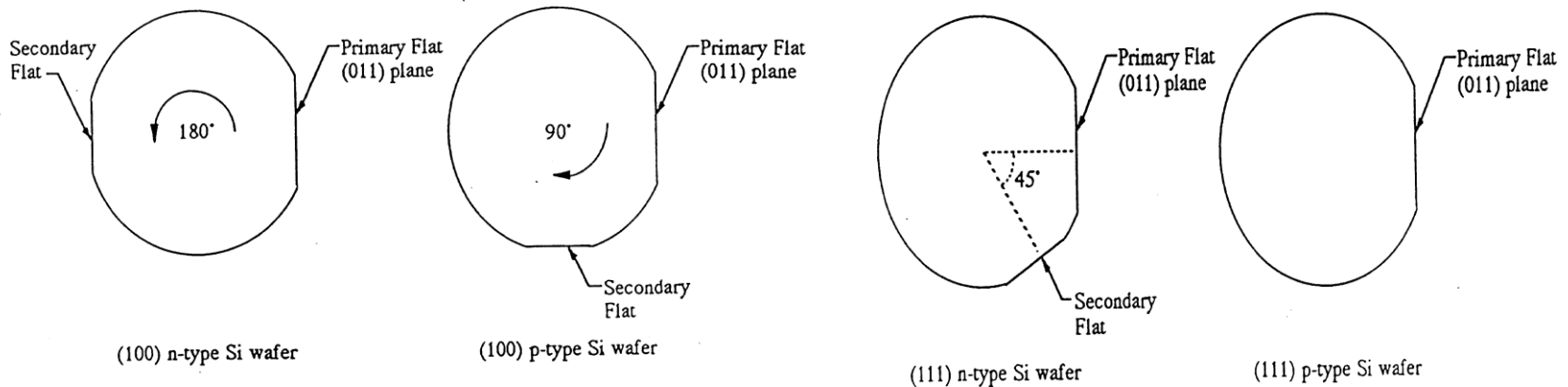


Figure 1(a). n and p-type Si wafers and associated flats for (100) oriented surfaces.

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Figure 1(b). Flat orientation for (111) Si wafers

1.3 Lattice type in crystalline solids

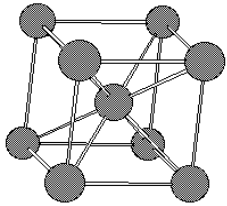
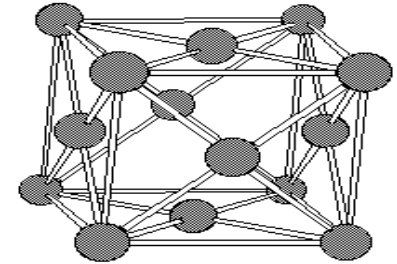


Figure 3(c). BCC
Unit Cell

Figure 3(b). Face
centered cubic (FCC)



Cubic structures are one of the most common structures for most of the semiconductors. For simplicity we will learn briefly about three types of cubic structures, along with these concepts packing factor is introduced. In these structures 'a' is called lattice constant which is equal in all three directions (i.e. $a = b = c$). **Lattice constant a** : Nearest distance between two atoms (center to center). **Number of atoms per unit cell is Z** :

Table 2. Most common Unit cells in semiconductors

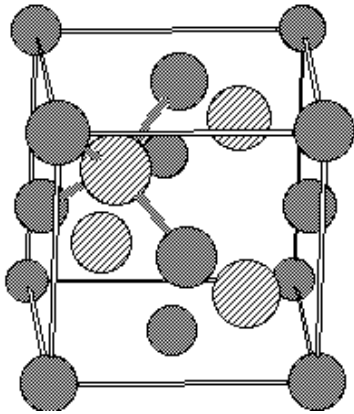


Fig. 3(d)Diamond/Zinc Blende

Binary: GaAs, ZnS, Ternary: InGaAs, ZnSSe, ZnCdSe, InGaN

Quaternary: InGaAsP,

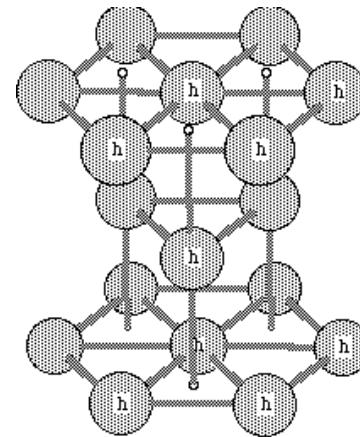


Fig.3(e). hexagonally
closed packed, HCP

1.4. Crystallographic Planes and Directions: Miller Indices

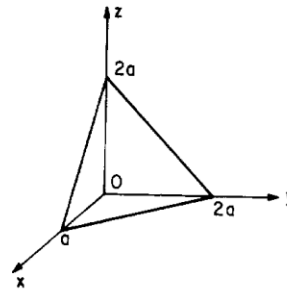


Fig. 4 A (211)-crystal plane.

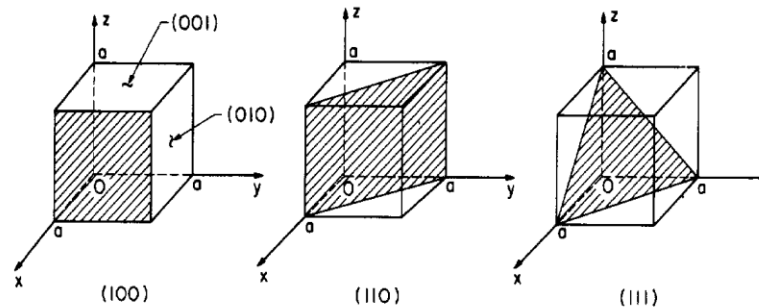


Figure 4(a). Illustration of a
lattice plane ABC.

Figure 6. (a) Diamond lattice (top) (b) Zinc blende lattice

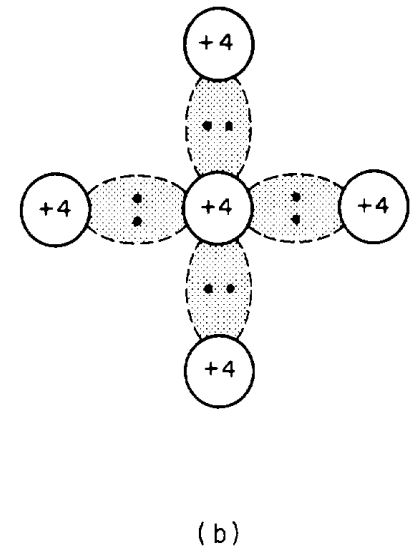
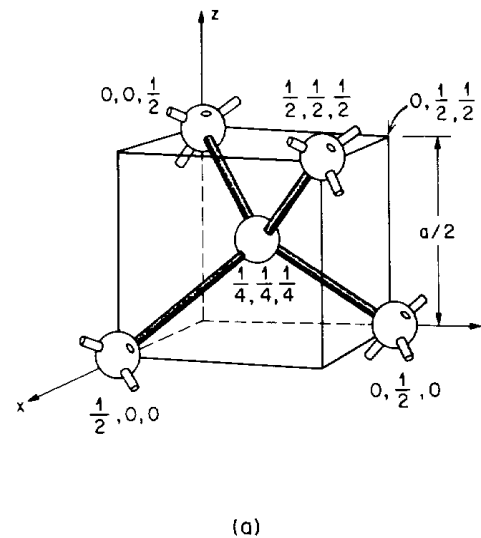
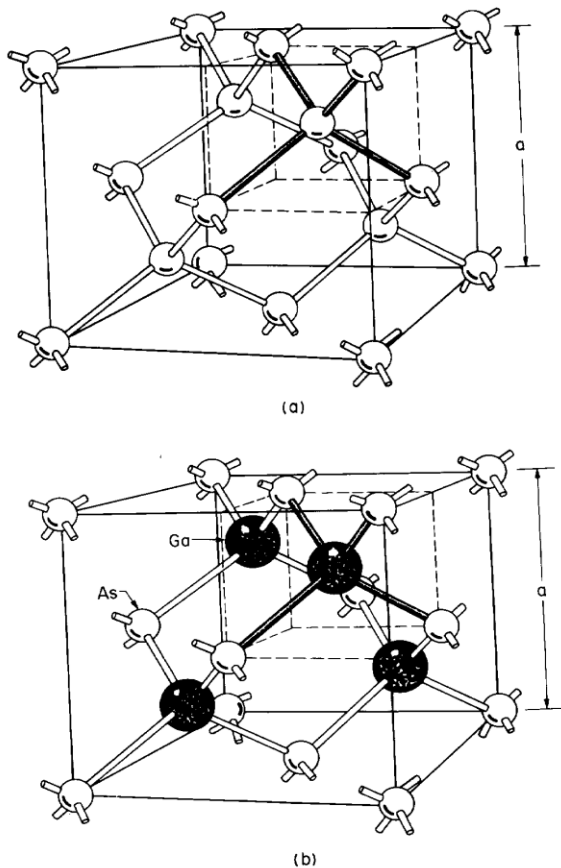


Figure 7. (a) A tetrahedral bond, (b) Schematic two-dimensional representation of a tetrahedral bonding.

s-p³ hybridization: When atoms are brought together, that is in the process of forming a solid; we increase the interaction between carriers. This is depicted by the lowering of the potential energy (as shown for sodium atoms). In the case of Si solid, the interaction between 3s and 3p electron takes place. This leads to hybridization, which in turn results in lowering of the energy as compared to the free atoms. Therefore, the interaction between s orbital and p orbital leads to sp³ hybridization. This is shown in Fig. 15(a).

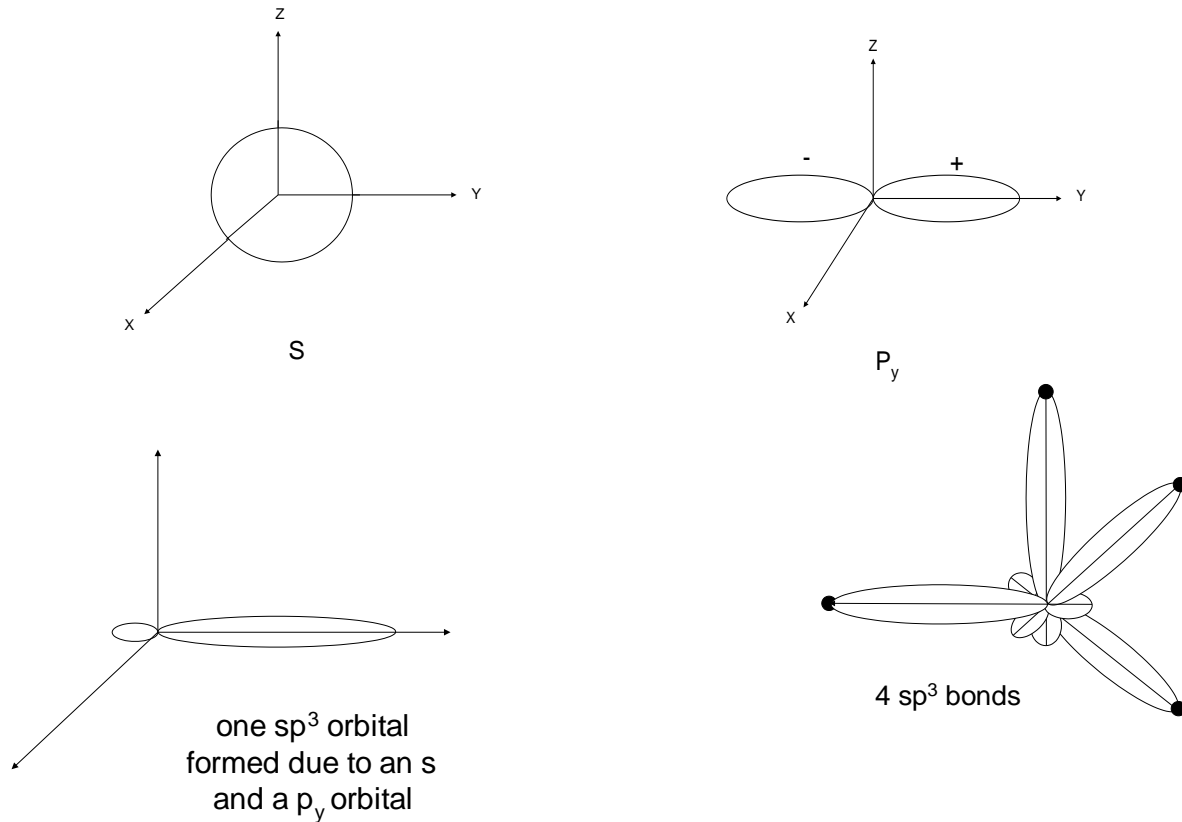


Fig. 15(a). Schematic representation of s, p_y, sp³, and tetrahedral orbitals.

1.5 Chemical Bonds and Energy Bands: Qualitative

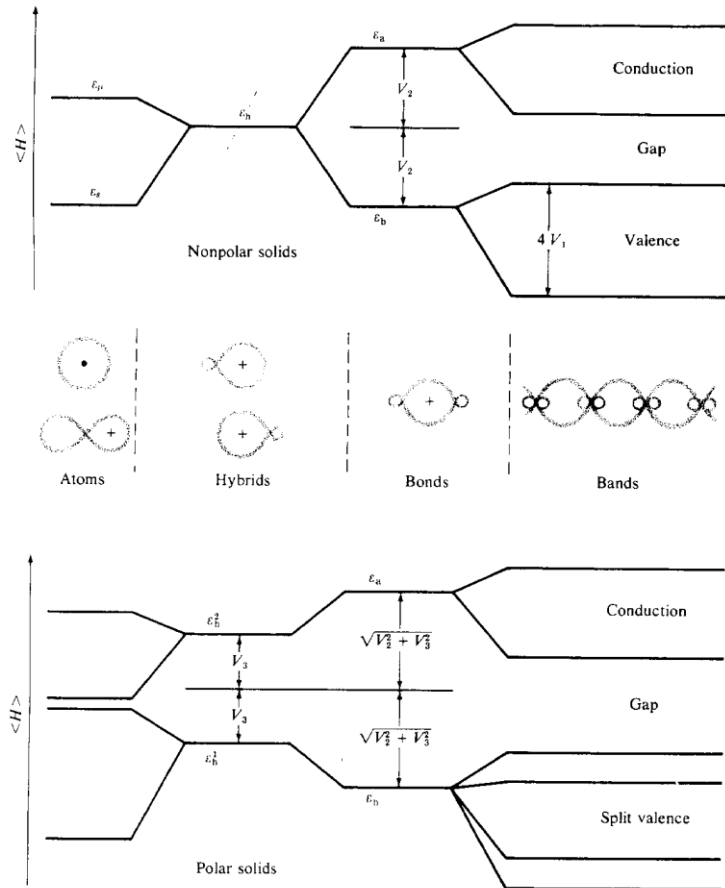


Figure 16. Successive transformations of linear combinations of atomic orbitals

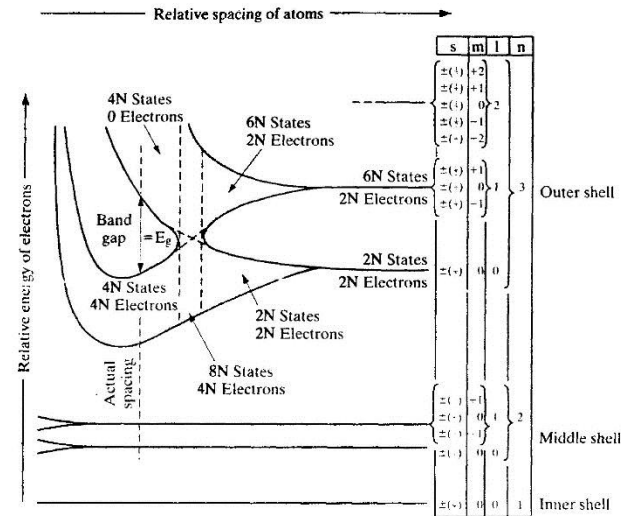
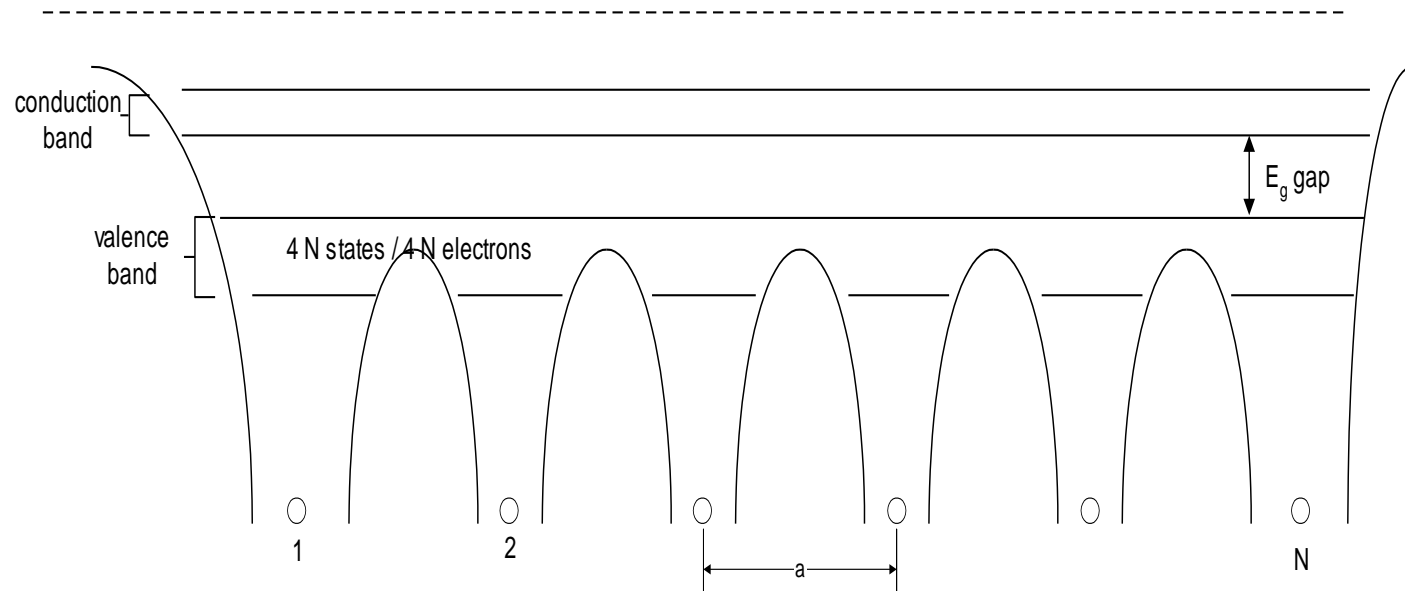


Figure 3-3 Energy levels in Si as a function of inter-atomic spacing. The core levels ($n = 1, 2$) in Si are completely filled with electrons. At the actual atomic spacing of the crystal, the $2N$ electrons in the $3s$ sub-shell and the $2N$ electrons in the $3p$ sub-shell undergo sp^3 hybridization, and all end up in the lower $4N$ states (valence band), while the higher lying $4N$ states (conduction band) are empty, separated by a bandgap.

Fig. 17 Energy band formation from discrete levels. (Reference: B. Streetman)

Fig. 19. Valence and conduction bands separated by energy band gap E_g .



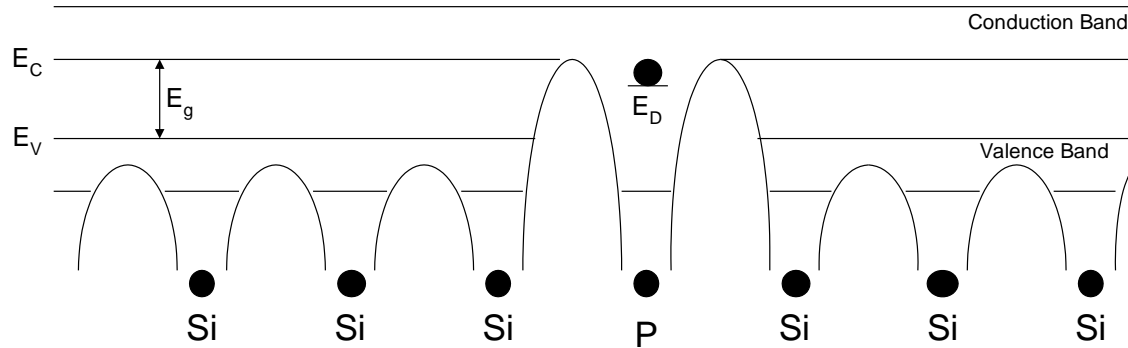


Fig. 26. Location of donor level E_D with respect to conduction band

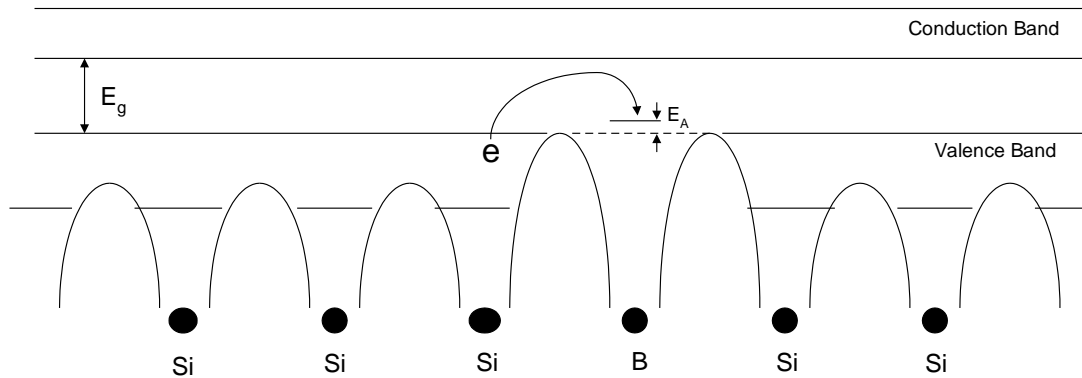


Fig. 27. Location of an acceptor level with respect to valence band

1.7 Doping of Si: Donor and Acceptor impurities

Pure Si crystal is called intrinsic and it has equal concentrations of electrons (n) and holes (p).

Addition of certain type of impurities makes semiconductors behave as p or n-type. This doping of impurities makes the semiconductor extrinsic or doped. In silicon, if we introduce elements from IIIrd column, we obtain p-type behavior, and Vth group elements make it n-type Si. Similarly, GaAs can be doped n-type by replacing Ga by IVth group elements. Alternatively, As can be substituted by VIth group elements to serve as donors.

II	III	IV	V	VI
Zn	B	C	P	S
Cd	Al	Si	As	Se
	Ga	Ge	Sb	Te

If N_D is the number of phosphorus atoms that are added per unit volume in Si, the electron and hole concentrations are (in approximation that assumes that all atoms are ionized):

$$n \approx N_D \text{ (Donors)} \quad (32)$$

$$p = n_i^2 / N_D \quad (33)$$

1.8 Carrier Concentrations in a semiconductor

Electrons in the conduction band fill first the lowest energy states and then the higher energy states. The concentration dn between E and $E+dE$ energy in the conduction band is expressed by

$$dn(E) = N(E) f(E) d(E). \quad (36)$$

Here, $N(E)$ is the density of energy levels (or states) and $f(E)$ is the statistical distribution function. Function $f(E)$ gives us the probability of finding electrons at an energy state E . There are two statistics used for electrons:

- (1) Maxwell-Boltzmann and
- (2) Fermi-Dirac.

Fermi-Dirac statistics under certain concentrations results in Maxwell-Boltzmann distribution. As a result we will use the Fermi-Dirac distribution function which covers all ranges of doping and carrier concentrations. [see summary at the end of Chapter 1 about various statistics].

Where E_f is Fermi Energy

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}} \quad (37)$$

The electron concentration ‘ n ’ accounting for the entire conduction band

$$n = \int_0^{\infty} N(E) f(E) dE \quad (38)$$

The hole concentration ‘ p ’ accounting for the entire valence band

$$p = \int_{-E_g}^{-\infty} N(E) [1 - f(E)] dE \quad (39)$$

Carrier Transport: Drift and Diffusion

$$\text{Drift Current: } I_n = J_n A = - (q \mu_n n) E A$$

$$\text{Diffusion Current density: } J_n = +q D_n \nabla n, [\text{Fick's Law}]$$

$$\text{Total current} = \text{Diffusion Current} + \text{Drift Current}$$

$$\text{Einstein's Relationship: } D_n / \mu_n = kT / q$$

$$n_{no} p_{no} = n_i^2 \quad \text{n-Si} \quad N_D = N_n = n_{no}$$

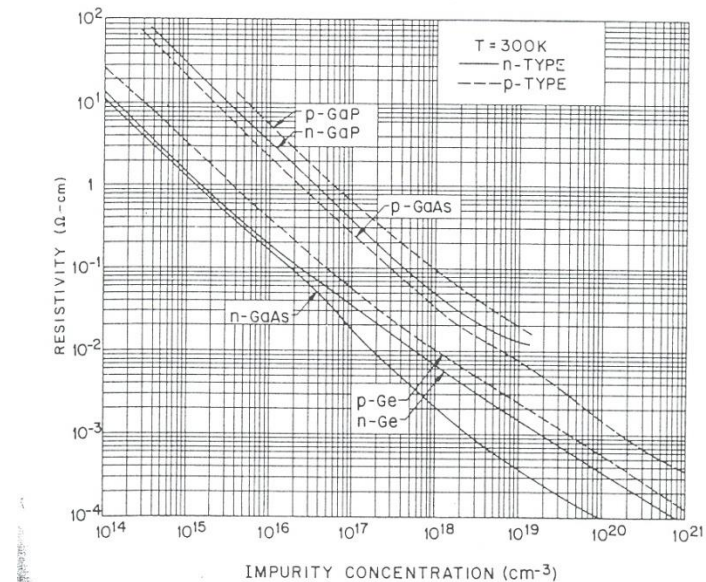
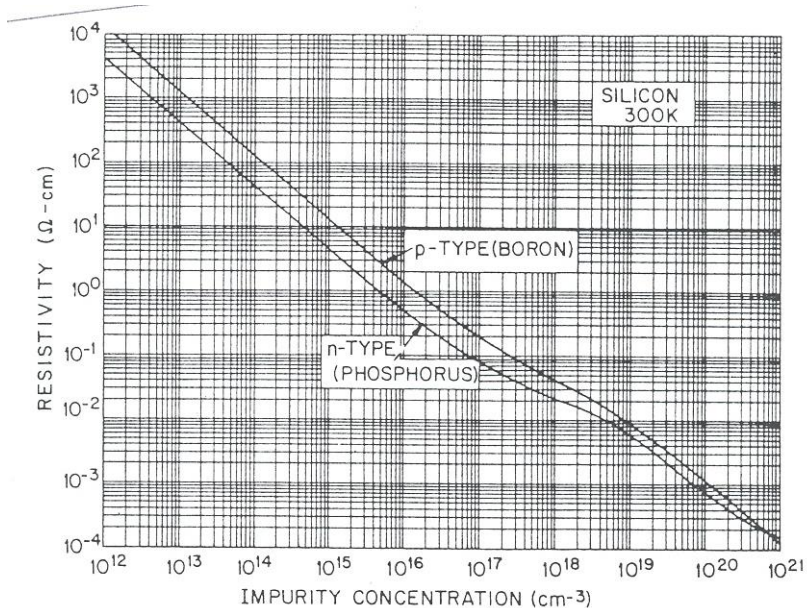
$$p_{po} n_{po} = n_i^2 \quad \text{p-Si} \quad N_A = P_p = P_{po}$$

Conductivity σ , Resistivity $\rho = 1/\sigma$

Current density J in terms of conductivity σ and electric field E : $J = \sigma E = \sigma (-\nabla V) = -\sigma \nabla V$

$$I = J A = \sigma E (W d),$$

In n-type Si, $\sigma_n = q \mu_n n_{no} + q \mu_p p_{no}$



Drift and Diffusion of holes in p-Si

In p-type Si,

The conductivity is: $\sigma_n = q \mu_p p_{po} + q \mu_n n_{po}$

Drift Current: $I_{p \text{ drift}} = J_p A = (q \mu_p p) E A$

Diffusion Current density: $J_p = -q D_p \nabla p$, [Fick's Law]

Diffusion current: $I_{p \text{ diff}} = -q A D_p \nabla p$

Einstein's Relationship: $D_n/\mu_n = kT/q$

Total hole current = Diffusion Current + Drift Current

$$I_p = -q A D_p \nabla p + (q \mu_p p) E A$$

Carrier concentration

When a semiconductor is pure and without impurities and defects, the carrier concentration is called **intrinsic concentration** and it is denoted by n_i . When electron concentration n is equal to hole concentration p , we have an intrinsic semiconductor.

$$n = p = n_i.$$

n_i as a function of temperature, see Figure 17 (page 28) and Fig. 11 (page 69).

Also, n_i can be obtained by multiplying n and p expressions (page 68 of notes)

$$n_i^2 = 4 \left[\frac{4 \pi^2 m_n m_p (kT)^2}{h^4} \right]^{3/2} e^{\frac{-E_g}{kT}}$$
$$n_i = 2 \left[\frac{2 \pi kT}{h^2} \right]^{3/2} (m_n m_p)^{3/4} e^{\frac{-E_g}{2kT}}$$

Carrier Concentration in Doped (Extrinsic) Semiconductors: Doped n- and p-type

IIIrd or Vth group elements in Si and Ge are used to dope them to increase their hole and electron concentrations in n-type and p-type, respectively, .

Vth group elements, such as Phosphorus, Arsenic, and Antimony, have one more electron in their outer shell, as a result when we replace one of the Si atoms by any one of the donor, we introduce an extra electron in Si.

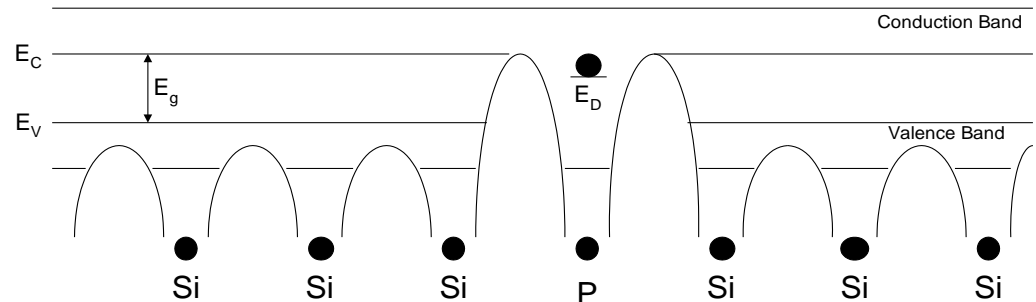
These Vth group atoms are called as donors. Once a donor has given an electron to the Si semiconductor, it becomes positively charged and remains so.

Whether a donor atom will donate its electron depends on its ionization energy E_D . If there are N_D donor atoms per unit cm^3 , the number of the ionized donors per unit volume is given by Eq. 93

$$N_D^+ = N_D \left[1 - \frac{1}{1 + \frac{1}{2} e^{\frac{(E_D - E_f)}{kT}}} \right]$$

Note that there is an additional factor of $\frac{1}{2}$ in Eq. 93. This is because when a donor atom donates the electron, it cannot occupy another electron despite its spin.

Fig .26



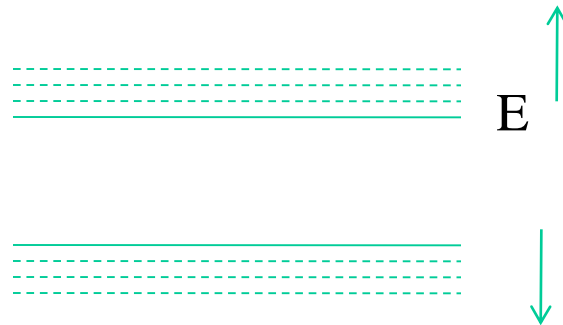
Fermi-Dirac Statistics

We have used a statistical distribution function, which tells the probability of finding an electron at a certain level E . This statistics is called Fermi-Dirac statistics, and it expresses the probability of finding an electron at E as Eq. (72)

$$f_{FD}(E) = \frac{1}{1 + e^{(E - E_f / kT)}}$$

E_f is the energy at which the probability of finding an electron is $\frac{1}{2}$ or 50%.

In brief, donor doped semiconductors have more electrons than holes.



Acceptors and p-type semiconductors:

We can add IIIrd group elements such as Boron, Indium and Gallium in Si. When they replace a Si atom, they cause a deficiency of electron, as they have three electrons in their outer shell (as compared to 4 for Si atom).

These are called acceptor atoms as they accept electrons from the Si lattice which have energy near the valence band edge E_v . Eq. 101 expresses the concentration of ionized acceptor atoms.

$$(101) \quad N_A^- = N_A * \frac{1}{1 + \frac{1}{4} e^{\frac{(E_A - E_f)}{kT}}}$$

N_A^- is the concentration of the ionized acceptor atoms that have accepted electrons. E_A is the empty energy level in the acceptor atom.

N_A^- is the concentration of ionized acceptors that have accepted an electron from the valence band top. Here, a factor of $\frac{1}{4}$ is due to degeneracy of valence band (heavy hole and light hole bands at the top of the valence band, and $\frac{1}{2}$ due to spin factor.

Hole conduction in the valence band: The electron, which has been accepted by an acceptor atom, is taken out of the Si lattice, and it leaves an empty energy state behind. This energy state in turn is made available to other electrons. It is occupied by other electrons like an empty seat in the game of musical chairs. This constitutes hole conduction.

Donors and acceptors in compound semiconductors

Semiconductors such as GaAs and InGaAs or ZnMgSSe are binary, ternary, and quaternary, respectively. They represent III-V and II-VI group elements.

For example, the doping of GaAs needs addition of group II or VI elements if we replace Ga and As for p and n-type doping.

In addition, we can replace Ga by Si for n-type doping. Similarly, if As is replaced by Si, it will result in p-type doping. So Si can act as both n and p-type dopant depending which atom it replaces.

Whether Si is donor or acceptor depends on doping temperature.

1.9.2.1. Electron and hole concentration in n-doped semiconductor

(a) *Exact procedure* to find n and p using charge neutrality condition

Charge neutrality condition in n-type semiconductor can be written as:[Eq. 91 on page 44] $qN_D^+ + qp = qn$ (91)

$$qN_D^+ + qp_{no} = qn_{no}, (92) \quad n_{no} = n_n$$

$$N_D \left[1 - \frac{1}{1 + \frac{1}{2} e^{\frac{(E_D - E_f)}{kT}}} \right] + 2 \left[\frac{2\pi m_p kT}{h^2} \right]^{\frac{3}{2}} \bullet e^{\frac{-(E_f + E_g)}{kT}} = 2 \left[\frac{2\pi m_n kT}{h^2} \right]^{\frac{3}{2}} e^{\frac{E_f}{kT}} \quad (94)$$

In Eq. 94, the first term is qN_D^+ , second term is hole concentration, and the right hand side is electron concentration. Hole and electron concentration will be derived later on.

In this equation, we know all parameters except E_f . One can write a short program and evaluate E_f or assume values of E_f and see which values makes left hand side equal to the right hand side.

(b) Approximate method to find electron and hole concentrations in n-type semiconductor (page 45)

Here, we start with the charge neutrality condition. $qN_D^+ + qp_{no} = qn_{no}$

We assume p_{no} or hole concentration $= n_i^2/N_D$. Substituting p_{no} in the charge neutrality equation, we get electron concentration by solving a quadratic equation

(96)

$$n_n^2 - N_D n_n - n_i^2 = 0$$

Its solution is:

$$n_n = \frac{1}{2} N_D + \sqrt{N_D^2 + 4n_i^2} \quad (98)$$

(c) Most simplified approximation to find electron and hole concentrations

Simple expressions for electron and hole concentrations in n-Si having N_D concentration of donors (all ionized). Electron concentration is $n = n_n$ or $n_{no} = N_D$, (here, the subscript n means on the n-side or in n-Si; additional subscript 'o' refers to equilibrium). Hole concentration is $p_{no} = n_i^2/N_D$.

For p-Si having N_A acceptor concentration (all ionized), we have $p_p = N_A$,
 and electron concentration $n_p = (n_i^2)/N_A$

Effect of Temperature on Carrier Concentration

The intrinsic and extrinsic concentrations depend on the temperature. For example, in Si the intrinsic concentrations at room temperature ($\sim 300\text{K}$) is $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. If you raise the temperature, its value increases exponentially (see relation for n_i).

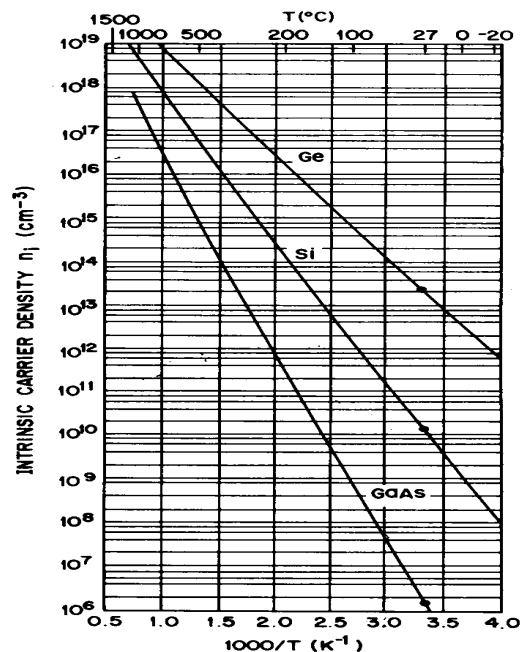


Fig. 11 Intrinsic carrier densities of Ge, Si, and GaAs as a function of reciprocal temperature. (After Thurmond, Ref. 20.)

Fig. 29. Intrinsic carrier concentration n_i as a function of T (Ref. #2).

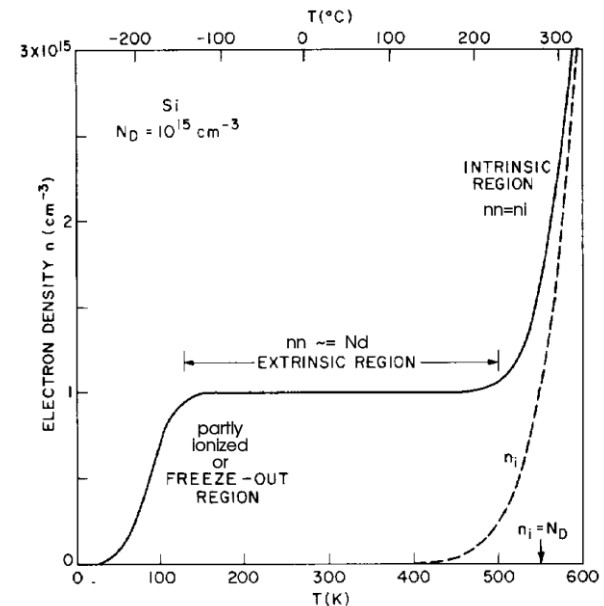


Figure 30. Temperature dependence of carrier concentration in an n-type Si.

1.8.4 Derivation of carrier concentration expression using density of states in conduction and valence band, and occupancy of electrons and holes using Fermi-Dirac distribution [page 38]

We started this method on page 14. The electron concentration in conduction band between E and $E+dE$ energy states is given by

$dn = f(E) N(E) dE$. To find all the electrons occupying the conduction band, we need to integrate the dn expression from the bottom of the conduction band to the highest lying level or energy width of the conduction band.

That is, $n = \int_0^{\infty} N(E) f(E) dE$ Eq. 76

This equation⁰ assumes that the bottom of the conduction band $E_c = 0$. Substituting for

$N(E)$ the density of states expression, $N(E)dE = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} E^{1/2} dE$ (58) $\hbar = \frac{h}{2\pi}$

$$n = \int_0^{\infty} \frac{1}{1 + \exp(E - E_f / kT)} \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2} \right)^{3/2} E^{1/2} dE$$

$$n = 2 \left[\frac{2\pi m_n kT}{h^2} \right]^{3/2} \cdot e^{\frac{E_f}{kT}} \quad (84A)$$

An alternate expression results, if E_c is not assumed to be zero.

$$n = 2 \left[\frac{2\pi m_n kT}{h^2} \right]^{3/2} \cdot e^{\frac{-(E_c - E_f)}{kT}} \quad (84B)$$

Another form is $n = n_i \exp[(E_f - E_i)/kT]$ (84C).

Hole concentration p

To find all the hole occupying the valence band, we need to integrate the dp expression from the top of the valence band to the lowest lying level or energy width of the valence band.

$$p = \int_{-\infty}^{E_v} N(E)[1 - f(E)]dE \quad (77)$$

This equation gets modified if valence band edge is $-E_g$ or E_C is our energy reference.

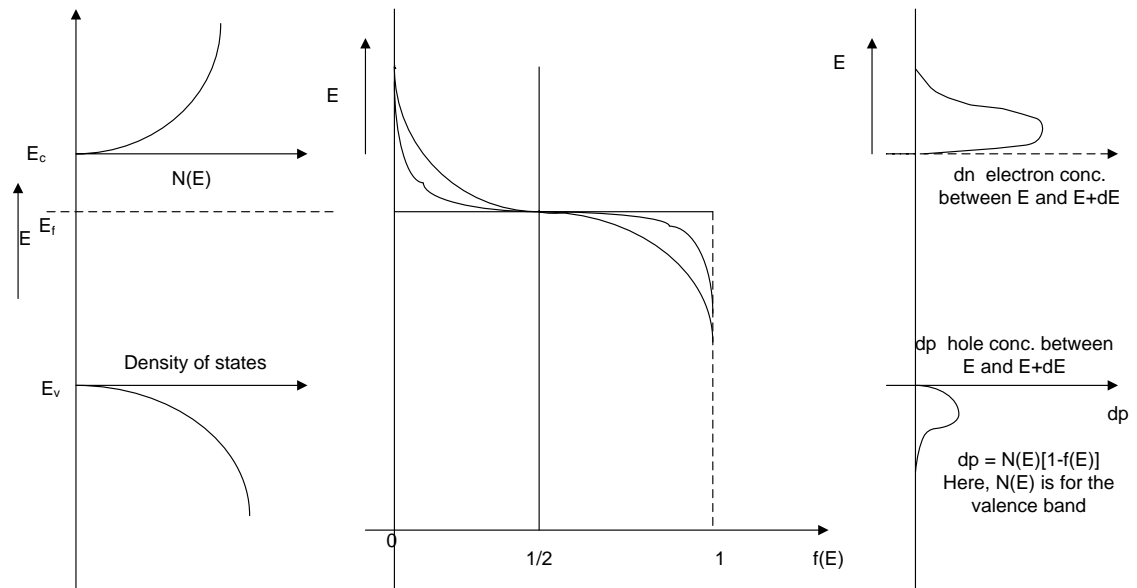
$$p = \int_{-\infty}^{-E_g} N(E)[1 - f(E)]dE \quad (78)$$

$$p = 2 \left[\frac{2\pi m_p kT}{h^2} \right]^{3/2} \cdot e^{-\frac{(E_f + E_g)}{kT}} \quad (85A) \quad \text{here } E_c=0 \text{ and } E_v=-E_g.$$

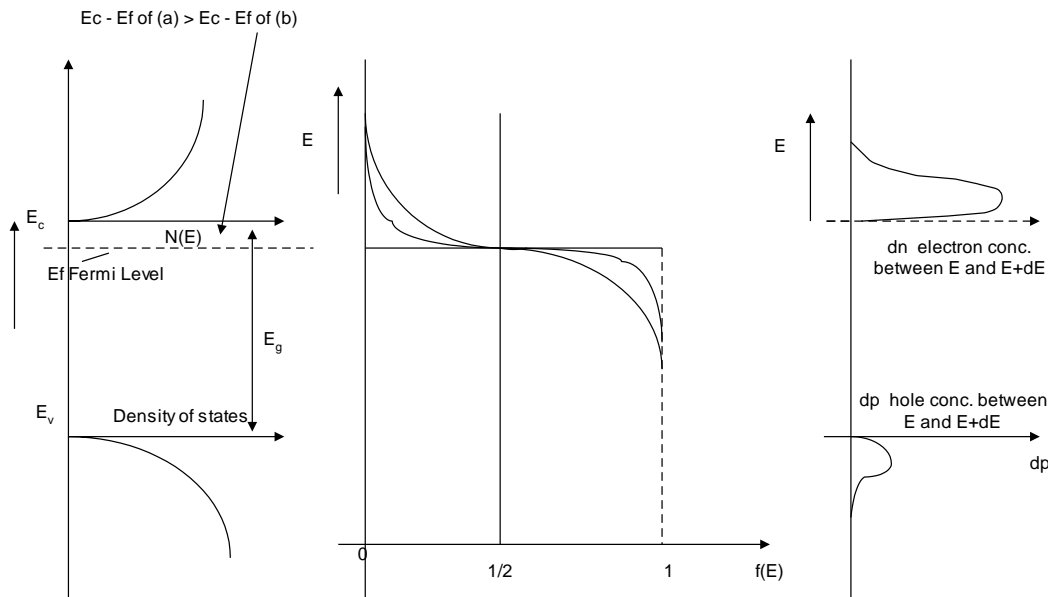
$$p = 2 \left[\frac{2\pi m_p kT}{h^2} \right]^{3/2} \cdot e^{-\frac{(E_f - E_v)}{kT}} \quad (85B)$$

Still another form of p is expressed in terms of n_i
 $p = n_i \exp[(E_i - E_f)/kT] \quad (85C),$

Graphical way to look at carrier concentrations, pp. 40-41



(a)



(b)

Figure 28(a) and 28(b) on the following page illustrate the relationship between carrier concentrations and location of the Fermi level with respect to the band edges. In Fig. 28(a) the energy separation ($E_c - E_f$) between the conduction band edge and the Fermi level E_f is greater than in Fig. 28(b). As a result there are more electrons in case (b), obtained by integrating the dn plot. If n is larger, it means hole concentration is smaller. Similarly, one can plot the situation, when E_f is near the valence edge, now because of the shift of E_f , dp plot will have a larger magnitude, yielding a greater value of hole concentration p than electrons.

Energy band diagram in semiconductors

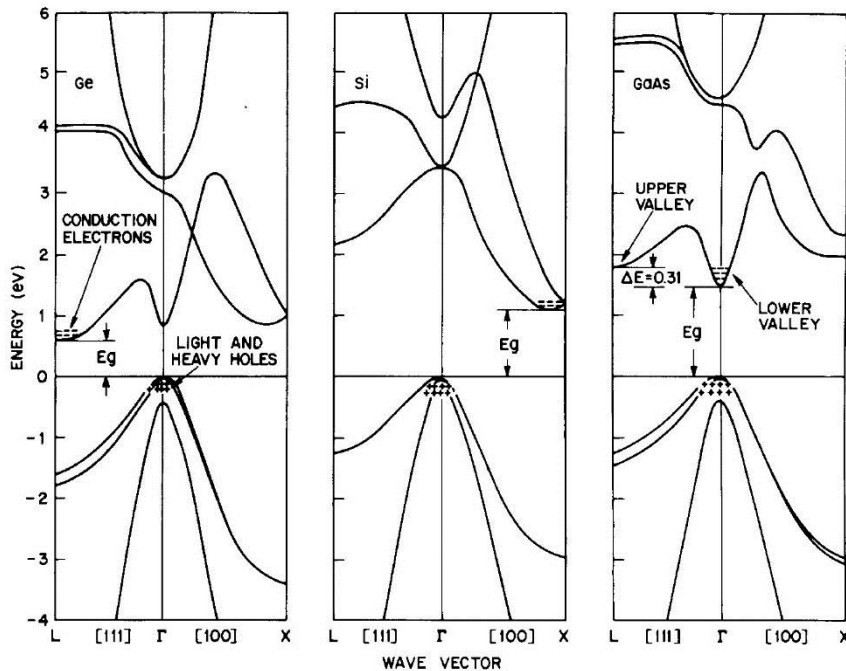


Fig. 25(a). Energy vs wave vector, E-k, band structure for germanium (left panel), silicon (middle), gallium arsenide. E_g is the energy gap.

Following information is obtained from an E-k diagram.

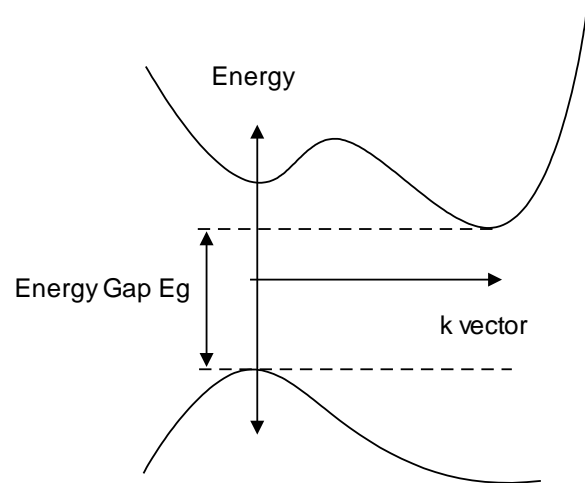
(a) Direct and indirect energy gap semiconductors

Examples of Direct Band Gap Semiconductors: GaAs, InP, ZnSe, GaN. Examples of Indirect Band Gap Semiconductors : Si, Ge, and GaP. Semiconductors are generally characterized by either as indirect energy gap or direct energy gap. This is shown in Fig. 25(b) on next page.

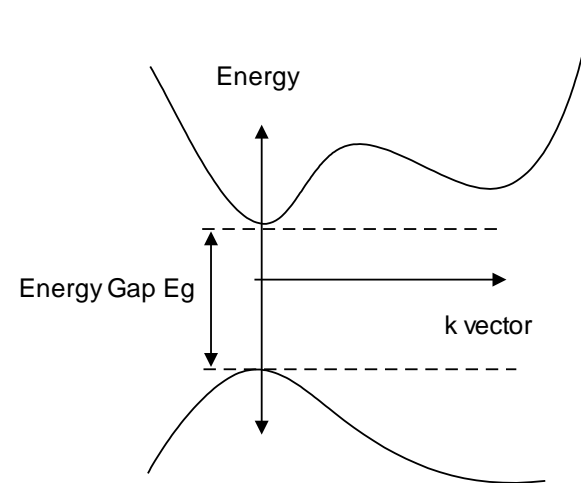
$$1/m^* = \frac{d^2 E}{\hbar^2 d^2 k}$$

(b) Effective mass of electrons and holes.

• Direct and Indirect Energy Gap Semiconductors



E-K diagram of an indirect semiconductor



E-K diagram of a direct semiconductor

Fig. 25b. Energy-wavevector (E-k) diagrams for indirect and direct semiconductors (page 29). Here, wavevector k represents momentum of the particle (electron in the conduction band and holes in the valence band). Actually momentum is $= (\hbar/2\pi)k$.

Table 4. Expressions for energy and momentum for electrons/holes, photons and phonons p. 62

	Electrons & Holes	Photons	Phonons
Statistics	F-D & M-B	Bose-Einstein	Bose-Einstein
Velocity	v_{th}, v_n $1/2 m v_{th}^2 = 3/2 kT$	Light c or $v = c/n_r$ n_r = index of refraction	Sound $v_s = 2865$ meters/s in GaAs
Mass	m_n, m_p (material dependent)	No mass	No mass
Energy	E-k diagram $E_{elec} = 25\text{meV to } 1.5\text{eV}$	ω -k diagram ($E = \hbar\omega$) $\omega \sim 10^{15}$ /s at $E \sim 1\text{eV}$ $E_{photons} = 1\text{-}3\text{eV}$	ω -k diagram ($E = \hbar\omega$) $\omega \sim 5 \times 10^{13}$ /s at $E \sim 30\text{meV}$ $E_{phonons} = 20\text{-}200$ meV
Momentum	$P = \hbar k$ $k = 2\pi/\lambda$ $\lambda = 2\pi v_{elec}/\omega$	momentum: 1000 times smaller than phonons and electrons	$P = \hbar k$ $k = 2\pi/\lambda$ $\lambda = 2\pi v_s/\omega$

Nanostructures: Energy level quantization and density of states

1.8.2 Density of states in Nanostructures

The density of states in semiconductors, when there is carrier confinement, is expressed in Table 3. Carrier confinement occurs in potential wells or quantum wells, quantum wires (nanotubes) and quantum dots.

Quantum well energy levels: The electrons and holes are confined in x direction via well. Electron energy levels in rectangular well with width = L_x

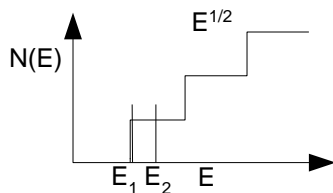
$$E_e = \frac{\hbar^2 \pi^2}{2m_e^*} \left(\frac{n_x^2}{L_x^2} \right) \quad (60)$$

. The hole energy level in a valance band (for heavy holes) is:
$$E_{hh} = \frac{\hbar^2 \pi^2}{2m_{hh}^*} \left(\frac{n_x^2}{L_x^2} \right) \quad (61)$$

Quantum Wires: The electron and hole energy levels are confined in x and y directions via potential barriers.

$$E_e = \frac{\hbar^2 \pi^2}{2m_e^*} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) \quad (62)$$

$$E_{hh} = \frac{\hbar^2 \pi^2}{2m_{hh}^*} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) \quad (63)$$



$$\sum_n \frac{m_e}{\pi \hbar^2 L_z} U(E - E_{nz}^e)$$

Density of states in quantum wells