Operating parameters:
Operating wavelength: green, red, blue, fiber optic wavelength 1.55 microns

Optical power output, expected external and wall conversion efficiency,

Operating structure
Cavity or Distributed Feedback type
Edge emitting or surface emitting.
Various laser structures

Fig. 28. An edge-emitting cavity laser with ridge.

Fig. 55 Cavity formed by one conventional mirror and one distributed Bragg reflector (DBR).

Fig. 56 In a vertical cavity surface emitting laser (VCSEL) the cavity thickness is multiple of half
Quantum Dot Structure

$$d = (50 \times 3) + (100 \times 2) + (150 \times 2) = 650 \text{Å} = 0.065 \text{mm}$$

D for dot and B for barrier.

Fig. 44 Active layer thickness: $d = (50 \times 3) + (100 \times 2) + (150 \times 2) = 650 \text{Å} = 0.065 \text{mm}$.
VCSELs (vertical cavity surface emitting lasers)

Fig. 57 Photograph of a 850nm GaAs VCSEL.

Fig. 58 Surface-emitting laser [Ref. J. Jahns et al, Applied Optics, 31, pp. 592-596, 1992].
Conditions of Lasing:

\[ \gamma = n_r k_o - j \kappa k_o \quad (1) \]

Here, \( n_r \) is the real part of the complex index of refraction, \( k_o \) is the freespace propagation constant of \( (2\pi / \lambda) \) and \( \kappa \) is the extinction coefficient (which is related to the absorption coefficient \( \alpha \) as \( \kappa = \alpha \lambda / 4\pi \)).

\[ \gamma = n_r k_o - j \frac{\alpha \lambda}{4\pi} k_o \quad (2) \]

Figure 1 shows the electric field strength at various locations during successive passes.

Figure 1. Electric field strength in a cavity of length L
General Conditions of Lasing:

\[ E_o = t_1 t_2 E_i e^{-j\gamma L} + t_1 t_2 r_1 r_2 E_i e^{-3j\gamma L} + (r_1 r_2)^2 t_1 t_2 E_i e^{-5j\gamma L} \]
\[ = t_1 t_2 E_i e^{-j\gamma L} [1 + r_1 r_2 e^{-2j\gamma L} + (r_1 r_2)^2 e^{-4j\gamma L} + \cdots] \]

Summing the series

\[ E_o = \frac{t_1 t_2 E_i e^{-j\gamma L}}{(1 - r_1 r_2 e^{-2j\gamma L})} \]

\[ \frac{E_o}{E_i} = \frac{t_1 t_2 e^{-j\gamma L}}{(1 - r_1 r_2 E^{-2j\gamma L})} \]

Where \( \gamma = n_r k_0 - j \frac{\alpha \lambda}{4\pi} k_0 \)

For a given \( E_i, E_o \) will approach infinite if

\[ 1 - r_1 r_2 e^{-2j\gamma L} = 0 \]

\[ 1 - r_1 r_2 e^{-2j \left[ n_r \frac{2\pi}{\lambda} + j \frac{(g-\alpha)}{2} \right] L} = 0 \]

\[ 1 = 1 e^{-j2\pi m}, m \text{ is an integer} \]
General Conditions of Lasing:

(a) Representing the amplitude/magnitude

\[ g = \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

(b) Phase condition

\[ e^{-j2\pi m} = e^{-\frac{4j\pi mL}{\lambda}} \]

\[ L = \frac{m\lambda}{2n_r} \]
Resonant Cavity: Condition I for Lasing

\[
\frac{dm}{d\lambda} = -\frac{2n_r L}{\lambda^2} + \frac{2L}{\lambda} \frac{dn_r}{d\lambda} = -\frac{2n_r L}{\lambda^2} \left( 1 - \frac{\lambda}{n_r} \frac{dn_r}{d\lambda} \right)
\]

\[
d\lambda = -\frac{\lambda^2}{2Ln_r} \left( 1 - \frac{\lambda}{n_r} \frac{dn_r}{d\lambda} \right)^{-1} dm
\]

\[
\Delta \lambda = \pm \frac{\lambda^2}{2Ln_r} \left( 1 - \frac{\lambda}{n_r} \frac{dn_r}{d\lambda} \right)^{-1}
\]

Figure 2. Cavity with parallel end faces
Emission spectrum high lighting cavity modes
Fig. 3, p. 343

Figure 3 shows the emission spectrum highlighting cavity modes (also known as the longitudinal or axial modes) for the GaAs laser diode.

**Conditions and Calculations: GaAs**

- \( \lambda = 0.85 \mu m \)
- \( n_r = 3.59 \)
- \( L = 1000 \mu m \)
- \( \Delta \lambda = 2.01 \ \text{Å} \)
Equivalent of population inversion in semiconductor lasers: Condition II for Lasing

This condition is based on the fact that the rate of stimulated emission has to be greater than the rate of absorption.

\[ B_{21} n_2 \rho(h \nu_{12}) > B_{12} n_1 \rho(h \nu_{12}) \]  \hspace{1cm} (23)

 Strictly speaking, the rate of stimulated emission is proportional to:

(i) the probability per unit time that a stimulated transition takes place \(B_{21}\)

(ii) probability that the upper level \(E_2\) or \(E_c\) in the conduction band is occupied

\[ f_e = \left( \frac{1}{1 + e^{\frac{E - E_{fn}}{kT}}} \right) \], \(E_{fn}\) = quasi-fermi level for electrons.  \hspace{1cm} (24)

(iii) joint density of states \(N_j(E=\hbar \nu_{12})\)

(iv) density of photons with energy \(\hbar \nu_{12}\), \(\rho(\hbar \nu_{12})\)

(v) probability that a level \(E_1\) or \(E_v\) in the valence band is empty (i.e. a hole is there)

\[ f_h = \left( 1 - \frac{1}{1 + e^{\frac{E - E_{fpv}}{kT}}} \right) \]  \hspace{1cm} (25)

The rate of stimulated emission:

\[ = B_{21} \left[ \frac{1}{1 + e^{\frac{E - E_{fp}}{kT}}} \right] * N_j(E = h \nu_{12}) * \rho(h \nu_{12}) * \left[ 1 - \frac{1}{1 + e^{\frac{E - E_{fp}}{kT}}} \right] \]  \hspace{1cm} (26)

Similarly, the rate of absorption:

\[ = B_{12} \left[ \frac{1}{1 + e^{\frac{E - E_{fp}}{kT}}} \right] * N_j(E = h \nu_{12}) * \rho(h \nu_{12}) * \left[ 1 - \frac{1}{1 + e^{\frac{E - E_{fp}}{kT}}} \right] \]  \hspace{1cm} (27)
Using the condition that the rate of stimulated emission > rate of absorption; (assuming $B_{21} = B_{12}$), simplifying Equation (26) and Equation (27)

\[
\frac{l}{1 + e^{\frac{E_c - E_{fn}}{kT}}} \times \left[ 1 - \frac{l}{1 + e^{\frac{E_v - E_{fp}}{kT}}} \right] > \frac{l}{1 + e^{\frac{E_v - E_{fp}}{kT}}} \times \left[ 1 - \frac{l}{1 + e^{\frac{E_c - E_{fn}}{kT}}} \right]
\]

(28)

Further mathematical simplification yields if we use

\[ E_c - E_v = h\nu_{12} = h\nu \]

(29)

\[ E_{fn} - E_{fp} > E_c - E_v \]

(30)

Bernard - Douraffourg Condition \[1\]

\[ E_{fn} - E_{fp} > h\nu \]

(31)

Equation (31) is the equivalent of population inversion in a semiconductor laser. For band to band transitions \[ h\nu \geq E_g \]

\[ E_{fn} - E_{fp} > h\nu > E_g \]

(32)
Definition of quasi Fermi-levels

\[ n = n_i e^{\frac{E_{fn} - E_i}{kT}} \quad ; \quad n = N_e e^{\frac{E_{fn}}{kT}} \]
\[ p = n_i e^{\frac{E_i - E_{fp}}{kT}} \quad ; \quad p = N_V e^{\frac{E_{fp} - E_g}{kT}} \]

Multiply Equations (a) & (b),

\[ np = n_i^2 e^{\frac{E_{fn} - E_g}{kT}} \]

or

\[ n_e = n_i e^{\frac{E_{fn} - E_g}{2kT}} \]

\[ n_e |_{\text{minium}} = n_i e^{\frac{E_{fn}}{2kT}} = 10^7 \times e^{\frac{1.43}{240.0259}} = 9.75 \times 10^{18} \text{ cm}^{-3} \]

**Gain coefficient g and Threshold Current Density J_{th}**

The gain coefficient g is a function of operating current density and the operating wavelength \( \lambda \). It can be expressed in terms of absorption coefficient \( \alpha(h\nu_{12}) \) involving, for example, band-to-band transition.

\[ g = -\alpha_o (1 - f_e - f_h) \]
Derivation of $J_{TH}$

Rate of stimulated emission

$$ = B_{21} f_e N_j (E = h\nu) \rho(h\nu_{12}) f_h \quad (\text{Where } \rho(h\nu_{12}) = P \nu_v h_v \Delta \nu_s) $$

$$ = B_{21} f_e f_h (\alpha \gamma_g) n_v N_v \Delta \nu_s $$

Rate of absorption

$$ = (1 - f_h)(1 - f_e) \alpha \gamma_g n_v N_v \Delta \nu_s B_{12} $$

Net rate = Stimulated – absorption

$$ = [f_e f_h - (1 - f_e)(1 - f_h)] \alpha \gamma_g n_v N_v \Delta \nu_s B_{21} $$

$$ = -[1 - f_e f_h] \alpha \gamma_g n_v N_v \Delta \nu_s B_{21} \quad \text{and also note that } B_{12} = B_{21} $$

The gain coefficient is

$$ g = -\alpha (1 - f_e - f_h) $$

Rate of spontaneous emission

$$ R_v = r_v \Delta \nu_s = f_e f_h (\alpha \gamma_g) N_v \Delta \nu_s $$ (34)
where:
\( \alpha_o v_g \) = probability of absorbing a photon
\( N_v \) = number of modes for photon per unit frequency interval
\( \Delta v_s \) = width of the spontaneous emission line

Equation (34) gives

\[
\alpha_o = \frac{\frac{r_v \Delta v_s}{f_e f_h v_g N_v \Delta v_s}}{(35)}
\]

The total rate of spontaneous emission

\[
R_c = \int r_v d\nu \equiv r_v \Delta v_s
\]

(36)

\[
R_c = \frac{J}{q} \eta \frac{l}{d} = \frac{I}{A \cdot d} \cdot \frac{l}{q} \eta
\]

(37)

where:
\( R_c \) = Rate per unit volume
\( \eta \) = quantum efficiency of photon (spontaneous) emission
\( d \) = active layer width
\( A \) = junction cross-section

Equations (33), (35), (36) and (37) give

\[
g = \frac{\left( \frac{J}{q} \right) \eta \left( \frac{l}{d} \right) \left( f_e + f_h - 1 \right)}{f_e f_h v_g N_v \Delta v_s}
\]

(38)
\[ N_v = \frac{8\pi n_r^2 v^2}{c^2 v_g} \]  

(39)

\[ \frac{f_e + f_h - 1}{f_e f_h} = 1 - e^{-\frac{h\nu(E_g - E_p)}{kT}} = z(t) \]  

(40)

Substituting for \( N_v \) and \( \frac{f_e + f_h - 1}{f_e f_h} \), we get

\[ g = \frac{J \eta}{q d v_g \Delta v_s} \frac{c^2 v_g}{8\pi n_r^2 v^2} \left[ 1 - e^{-\frac{h\nu - \Delta \zeta}{kT}} \right] \]  

(41)

\[ \Delta \zeta = E_{fn} - E_{fp} \]

The condition of oscillation, Equation (15), gives

\[ g = \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \]  

(15)
Threshold current density

Using Equation (15) and Equation (42)

$$J_{th} \eta = \frac{c^2 v_s}{qd \nu s \Delta \nu_s 8 \pi n^2 v^2 \left[ 1 - e^{\frac{\nu s \Delta \nu_s}{kT}} \right]} = \alpha + \frac{l}{2L} \ln \frac{l}{R_1 R_2}$$

(43)

$$J_{th} = \frac{8 \pi \eta c^2 v^2 qd \Delta \nu_s}{\Gamma \eta c^2 z(T)} \left[ \alpha + \frac{l}{2L} \ln \frac{l}{R_1 R_2} \right]$$

(44)

When the emitted stimulated emission is not confined in the active layer thickness d, 
\Gamma, confinement factor is fraction of laser power in the active layer d divided by total power generated. 
$$\Gamma = 2 \pi^2 d^2 (n_{active}^2 - n_{clad}^2) / \lambda^2$$

\(z(T)\) depends on quasi Fermi levels which depend on electron and hole concentration sin active layer.

R1 and R2 are the reflectivity . (If you cannot find them, use R1=R2=0.3)

\(\eta\) is the quantum efficiency of photon production when electron and hole recombine.

\(\alpha\) is the absorption coefficient of light in the laser active layer. It is given.

Spontaneous line from active layer is \(\Delta \nu_s\)
Threshold current density

Example 3. Evaluate the threshold current density $J_{th}$ of a laser diode having an active layer thickness $d = 0.2 \, \mu m$, cavity length of $300 \, \mu m$ and cavity or contact width of $10 \, \mu m$. In addition, internal quantum efficiency $\eta_q = 0.9$, confinement factor $\Gamma = 0.5$, reflectivity of cavity facets $R_1 = R_2 = 0.3$, spontaneous emission line-width $\Delta \nu_s = 6.2 \times 10^{12} \, Hz$, $Z(T) \approx 0.8$. [Note that $Z(T)$ depends on quasi Fermi levels which depend on forward biasing current and the emitting photon energy.]

Solution: The current density is expressed as

$$J_{th} = \frac{8 \pi n_e^2 q d \Delta \nu_s}{\Gamma \eta_q Z(T)} \left( \frac{1}{\lambda^2} \left( \frac{\alpha}{2L} + \ln \left( \frac{1}{R_1 R_2} \right) \right) \right)$$

$$\frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) = \frac{1}{2 \times 200 \times 10^{-4}} \ln \frac{1}{0.3^2} = 60.2 \, cm^{-1}$$

$$J_{th} = \frac{8 \pi \times 3.59^2 \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-4} \times 6 \times 10^{12}}{0.8 \times 0.9 \times 0.5} \frac{1}{(0.84 \times 10^{-4})^2 \left( 20 + 60.2 \right)} = 196 A / cm^2$$
Laser Characteristics

5.4.1 Optical power-current characteristics: page 354
The laser diode emits spontaneously until the threshold current is exceeded (i.e. \( I > I_{\text{th}} \) Fig. 9).

Fig. 9  Power output versus current (P-I) characteristic.
Note that \( \Delta P_{\text{out}} \) is significant above \( \Delta I \) increments above \( I_{\text{th}} \). Fig. 10 shows the V-I characteristic.

Fig. 10. V-I characteristics of a laser diode.
P-I Behavior as a function of temperature
The $I_{th}$ value is a function of operating temperature. This is shown in Fig. 11.

![Graph showing P-I behavior as a function of temperature.]

Fig. 11: Variation of threshold current density as a function of temperature.

![Graph showing the temperature variation of threshold current.]

Figure 12: Temperature variation of threshold current.
Fig. 13. Far-field pattern of a AlGaAs-GaSs laser.

Fig. 14 (a) Beam divergence in transverse/perpendicular and (b) lateral directions.

\[ d < \frac{m\lambda}{2\sqrt{n_-^2}主动 - n_+^2} ] \text{ clad} \]

Eq. 73

\[ m=1 \text{ for single transverse mode} \]

\[ d=0.2\mu m \]

(a) Transverse direction (x)

\[ W < \frac{m\lambda}{2\sqrt{n_-^2}MiddleOfGainRegionInActiveLayer - n_+^2} ] CornerOfActiveLayerAwayFromWhareGuardsSmall \]

Eq. 74

m=1 for single lateral mode

(b) Lateral direction (y)
Single mode laser

What would you do to obtain single transverse mode and single lateral mode operation? Select \(d\) and \(W\) for single transverse mode and lateral mode, respectively.

**HINT:** Active layer thickness \(d\) to yield single transverse mode \(d < \frac{\lambda}{2(n_r^2-n_{r1}^2)^{1/2}}\), \(m=1, 2, 3,\)

Top contact (stripe) width \(W\) for single mode:\[W < \frac{\lambda}{2(n_{r,\text{center}}^2-n_{r,\text{corner}}^2)^{1/2}}\], \(m=1, 2;\)

if you cannot find it use \(W=5 \, \mu \text{m}\); Use \(n_{r,\text{center}} - n_{r,\text{corner}} \approx 0.005\) in gain guided lasers.

Cavity length \(L=500\) micron to star twith (You select \(L\) such that it will result in the desired \(J_{\text{TH}}\) and optical power output).
5.4 Power output, Power-Current (P-I), Near-field, Far-Field Characteristics

Power output of a laser is expressed as a fraction of estimated photon energy density generated accounting for the losses due to mirrors R1 and R2.

(Ref: A. Yariv: Optical Electronics, publisher HRW)

\[
P_{\text{out}} = \left( \frac{\text{Power emitted by stimulated emission in the cavity}}{q} \right) \times (\text{Fraction lost due to mirrors})
\]

\[
= \left[ \frac{(I - I_{TH}) \eta_{\text{inj}} \hbar \nu}{q} \right] \times \left[ \frac{\frac{1}{2L} \ln(1/R_1 R_2)}{\alpha + \frac{1}{2L} \ln(1/R_1 R_2)} \right]
\]

\[
\eta_{\text{int}} = \eta_{\text{inj}} \times \eta_q \quad \text{Just like LEDs}
\]

Assume \( \eta_{\text{inj}} \equiv 1, \eta_q = 0.9 \leftrightarrow 1.0 \)

External Differential Efficiency is defined as rate of change of power output per current over the threshold value.

\[
\eta_{\text{ex}} = \text{external differential} \rightarrow \text{quantum efficiency}
\]

\[
\eta_{\text{ex}} = \frac{d(P_{\text{out}}/h \nu)}{d[(I - I_{TH})/q]} = \text{Rate of increase of output power per increase in current}
\]

Power conversion efficiency

\[
\eta_c = \frac{P_{\text{out}}}{V_{\text{app}} \times I}
\]
Design specs: Lasing wavelength 980nm, threshold current density $J_{TH} < 1000A/cm^2$, single mode output power $P_{out} = 10$ mW.

Step A: Selection of Active layer, Substrate & Cladding Layers

We have to select proper active layer that would lase at 980nm, cladding layers that would provide adequate carrier and photon confinement.

$$h\nu = \frac{1.24}{0.98} = 1.265 eV$$

Bernard-Douraffour condition states that

$$E_{fi} - E_{fp} \geq h\nu \geq E_g$$

Let $E_g = 1.26$ eV

Find the composition from the following $E_g$ vs lattice constant chart. The active layer is In$_{0.12}$Ga$_{0.88}$As. (Lattice parameter = 5.7 Å)

Substrate: GaAs

InGaAs-GaAs lattice mismatch = $5.7 - 5.66 = 0.04 = \Delta a$

$$\frac{0.04}{5.66} = 0.00706 \Rightarrow \frac{\Delta a}{a_{GaAs}}$$

There is a 0.7% of mismatch.
Cladding layers:

(i) $E_g \bigg|_{\text{cladding}} - E_g \bigg|_{\text{Active}} = 0.2 - > 0.5 \text{eV}$

(ii) $n_r \bigg|_{\text{Active}} - n_r \bigg|_{\text{cladding}} = \Delta n_r \approx 0.01$

Index of refraction of $\text{In}_{0.12}\text{Ga}_{0.88}\text{As}$ using the index relation is given by,

$\text{In}_{x}\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y} => \text{In}_{0.12}\text{Ga}_{1-0.12}\text{As}$

$y = 1; \; x = 0.12$

$n_{rs} = 3.52xy + 3.39 (1-y) + 3.6y (1-x) + 3.56 (1-x) (1-y)$
$\quad = 3.52*0.12 + 0 + 3.6 (1-0.12)$
$\quad = 3.5904$

$n_r \bigg|_{\text{clad}} \leq 3.49$

Cladding material:

Select GaAs or $\text{Al}_x\text{Ga}_{1-x}\text{As}$. Let $x=0.2$, $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As} => E_g \bigg| = 1.58 \text{eV}$

$n_r \bigg|_{\text{AlGaAs}} = 3.59 - 0.71x + 0.091x^2$

$x = 0.2$

$n_r = 3.452$

So $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ satisfies the index condition. The energy gap is 1.58 eV.

$E_g \bigg|_{\text{clad, Al}_{0.2}\text{Ga}_{0.8}\text{As}} - E_g \bigg|_{\text{Active, In}_{0.12}\text{Ga}_{0.88}\text{As}} = 1.58 - 1.26 = 0.32 \text{eV}$
Step B: Doping Level of Active / Cladding Layers

The doping levels in cladding are crucial as they should be higher doping than the injection levels \( n_e \) in the active layer that would satisfy the Bernard-Douraffourg condition.

\[
p = n = n_i
\]

The concentration is obtained from Bernard-Douraffourg condition

\[
E_{fit} - E_{fp} \geq h\nu \geq E_g
\]

\[
n_e = n_i e^{(E_{fit} - E_{fp}) / 2kT}
\]

\[
n_p = p_i = p_{pi} e^{(E_{fit} - E_{fp}) / 2kT}
\]

\[
p/n_e = n_i^2, \quad e^{(E_i - E_{fit}) / kT}
\]

\[
n_i = n_{i0}, \quad p = p_i
\]

\[
n_{i0} > n_{i0} e^{(E_g / 2kT)}
\]

\[
n_e > n_i e^{(E_g / 2kT)}
\]

Once \( p_{pi}, n_{i0} \) are known, the doping concentrations are related as:

\[
\begin{align*}
N_A & > p_i \\
N_D & > n_i
\end{align*}
\]

\[
n_i = n_i e^{E_{fit} / 2kT} = n_i e^{E_{fp} / 2kT}
\]

\[
n_i(\text{In}_0.12\text{Ga}_0.88\text{As}) = 10^7 \text{ cm}^{-3}
\]

\[
n_i(\text{GaAs}) = 10^7 \text{ cm}^{-3}
\]

\[
n_i(\text{In}_0.12\text{Ga}_0.88\text{As}) = 10^7 e^{-1.43 - 1.26 / 2 \times 0.0259} = 2.66 \times 10^4 \text{ cm}^{-3}
\]
Step C: Active layer thickness $d$ for a Single Transverse Mode and Stripe width $W$ for a single lateral mode

a) Active layer thickness $d \approx 0.2\mu m$

For single transverse mode, condition for single transverse mode ($m = 1$) is

Gain coefficient high then the carrier concentration is low and index is high. This is shown Figs. 24 and 25.

$$d < \frac{m\lambda}{2\sqrt{n_{\gamma}^2 \left|_{\text{active}} - n_{\gamma}^2 \left|_{\text{clad}}\right.}}$$

$$< \frac{0.98\mu m}{2*0.9857} \quad \text{where} \quad m=1,2,3 \quad < 0.5\mu m$$

Fig. 24. Gain coefficient in active layer along lateral axis $y$.

Fig. 25 (a) structure, (b) carrier concentration and index of refraction in the active layer.
b) Given for a single lateral mode \( W = 15\mu m \)

\[
W < \frac{m\lambda}{2\sqrt{\left(\frac{n_r^2}{n_r} \right)_{\text{MiddleOfGainRegionInActiveLayer}} - \left(\frac{n_r^2}{n_r} \right)_{\text{CornerOfActiveLayerAwayFromWhereGainIsSmall}}}}
\]

(74)

Fig. 26. Lateral confinement due to index variation in the center and end regions of the active layer.

\( n_r \) (center) \(- n_r \) (end) \( \approx 0.005 \)

Index increases if carrier concentration decreases. This is known as Gain Guiding and is responsible for lateral confinement.

\[
W \leq \frac{0.98\mu m}{2 \times 0.005 \times 2 \times 3.59} \leq 13.8\mu m
\]

**Confinement Factor** \( \Gamma \)

The simplest empirical expression for AlGaAs-GaAs heterostructure laser is

\[
\text{Transverse Confinement Factor} = \Gamma_T = 1 - \exp \left( -C \Delta n_r d \right), \text{ here } C = 3 \times 10^6
\]

\[
= 1 - \exp \left( -3 \times 10^6 \times 0.138 \times 0.2 \times 10^{-4} \right)
\]

\[
= 1 - 2.5 \times 10^{-4} \approx 1
\]

Or, use the general relation
Step D: Calculation of threshold current density $J_{th}$

$$R_1 = R_2 = \left( \frac{n_r|_{\text{active}} - n_{air}}{n_r|_{\text{active}} + n_{air}} \right)^2$$

$$= \left( \frac{3.59 - 1}{3.59 + 1} \right)^2 = 0.318$$

We need the cavity length $L$. It determines the optical power output. We will assume a $L$ value and see if $P_{out} > 10$mW, if it is not we will increase $L$.

Let $L = 500\mu$m.

$$J_{th} = \frac{8\pi n_r^2 q d \delta \nu_s}{\Gamma n_q Z(T)} \left( \frac{1}{\lambda^2} \right) \left( \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \right)$$

$$\alpha = 20 \text{ cm}^{-1}$$

$$\frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) = \frac{1}{2 \times 500 \times 10^{-4}} \ln \frac{1}{0.318^2} = 22.9 \text{ cm}^{-1}$$

Substituting for $\Delta \nu_s$ and other parameters,

$$J_{th} = \frac{8 \pi \times 3.59^2 \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-4} \times 6 \times 10^{12}}{1 \times 0.9 \times 0.5} \frac{1}{(0.98 \times 10^{-4})^2} (20 + 22.9) = 61.7 A / cm^2$$

$$I_{th} = w \times L \times J_{th} = 15 \times 10^{-4} \times 500 \times 10^{-4} \times 61.7 = 4.62 mA$$
Step E: Finding the Operating Current and Voltage that would provide desired laser output

In this step, we will calculate the operating current and voltage that would provide 10mW optical power. We will also calculate power conversion efficiency $\eta_{sc}$ and external differential efficiency $\eta_{ex}$.

Optical power output = 10mW

\[ \eta_{int} = \eta_{sc} \times \eta_{ex} \approx 0.9 \]

Stimulated photons generated in the cavity per second * fraction of light exiting mirrors due to $R_1$ and $R_2$. Stimulated photons are related to current above threshold.

\[ \text{Power Output} = \frac{(I - I_{th})\eta_{int}h\nu}{q} \left( \frac{1}{2L} \ln \left( \frac{1}{R_1R_2} \right) \right) \]

\[ 10mW = (I - 4.62 \times 10^{-3}) \times 0.9 \times 1.26 \times Volts \]

\[ I = 4.62 \times 10^{-3} + \frac{42.9}{22.9} \times 10^{-2} \times \frac{22.9}{0.9 \times 1.26} = 21.12 \times 10^{-3} \text{ A} \]

Operating voltage is related by the I-V equation

\[ I = I_s \left( e^{\frac{qV_{app\text{\_ref}}}{kT}} - 1 \right) \]

Junction Area = $wL = 15 \times 10^{-4} \times 500 \times 10^{-4} \text{ cm}^2 = 0.75 \times 10^{-4} \text{ cm}^2$

\[ I_s \equiv \frac{qAD_n n_{p0}}{L_n} + \frac{qAD_p p_{n0}}{L_p} \equiv \frac{qAD_n n_{p0}}{L_n} \]

In the active layer $n_{p0} = \frac{n_i^2 (InGaAs)}{N_A (InGaAs)} = \frac{(2.66 \times 10^8)^2}{10^{16}} = 7.07 \text{ cm}^{-3}$
Assume $D_n = 50 \text{ cm}^2 / \text{s}$; $I_n = 10^{-8}$

$L_n = \sqrt{50 \times 10^{-8}} = 7.07 \times 10^{-4} \text{ cm}$

$I_z = \frac{1.6 \times 10^{-19} \times 0.75 \times 10^{-4} \times 50 \times 7.07}{7.07 \times 10^{-4}} = 6 \times 10^{-8} \text{ A}$

From $I = I_z \left( e^{\frac{\eta_{\text{applied}}}{3T}} - 1 \right)$, $V_{\text{applied}} = \frac{kT}{q} \ln \left( \frac{I + I_z}{I_z} \right)$

$V_{\text{applied}} = 0.0259 \times \ln \left( \frac{21.12 \times 10^{-3} + 6 \times 10^{-12}}{6 \times 10^{-12}} \right) = 0.927 \text{ volts}$

**Conversion Efficiency:**

$\eta_c = \frac{10 \text{ mW}}{21.12 \times 10^{-3} \times 0.927} = 0.51 = 51\%$

**External Differential Efficiency:**

$\eta_{\text{ext}} = \frac{d(P_{\text{out}} / h\nu)}{d((I - I_{\text{th}}) / q)}$

Rewriting the $P_{\text{out}}$ expression gives,

$\left( \frac{P_{\text{out}} / h\nu}{q} \right) = \frac{(I - I_{\text{th}})}{q} \times \eta_{\text{int}} \times \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$

$\frac{\partial(P_{\text{out}} / h\nu)}{\partial((I - I_{\text{th}}) / q)} = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \eta_{\text{int}} \times \frac{1}{\alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)}$

$\eta_{\text{ext}} = 0.9 \times \frac{22.9}{42.9} = 0.48 = 48\%$
**Step F: Computation of Mode Separation** \( \Delta \lambda \), **frequency separation** \( \Delta \nu \), **beam divergence** in the junction plane \( \theta_{\parallel} \) and perpendicular to the junction plane \( \theta_{\perp} \).

\[
\Delta \lambda = \pm \frac{\lambda^2}{2L n_r} \left( 1 - \frac{\lambda}{n_r} \frac{dn_r}{d\lambda} \right)^{-1}
\]

\[0.98 \mu m = 0.98 \times 10^{-4} \text{ cm} \]

\[(0.98) \frac{\mu m}{(2 \times 500 \times 3.59) \mu m} \]

\[= 8.42 \times 10^{-4} \mu m \]

\[= \frac{(0.98 \times 10^{-4})^2}{2 \times 500 \times 10^{-4} \times 3.59} \left( 1 - \frac{0.98 \mu m}{3.59} \times 2.5 \mu m^{-1} \right) \]

\[= 8.42 \times 10^{-8} \text{ cm} = 8.42 \text{ Å} \]

\[
\Delta \nu = \frac{c \Delta \lambda}{\lambda^2} = \frac{3 \times 10^{10} \times 8.42 \times 10^{-8}}{(0.98 \times 10^{-4})^2} = 2.6 \times 10^{11} \text{ Hz} = 260 \text{ GHz} \quad (\because \lambda_1 \lambda_2 \approx \lambda_2; \lambda_1 = \lambda_2 + \Delta \lambda)
\]

**Beam Divergence:**

\[
\theta_{\parallel} = \frac{\lambda}{w} = \frac{0.98 \times 10^{-4}}{15 \times 10^{-4}} = 0.0653 \text{ radians} = 3.74^\circ
\]

\[
\theta_{\perp} = \frac{20x d}{\lambda} = \frac{20 \times 0.2 \times 0.2 \times 10^{-4}}{0.98 \times 10^{-4}} = 0.888 \text{ radians} = 50.9^\circ \quad (\because x = 0.2 \text{ for } Al_{0.2}Ga_{0.8}As)
\]

Where \( \frac{20x d}{\lambda} = \text{AlGaAs} \)

Alternate expression \( \theta_{\perp} = (4.0(n_{\text{activ.}}^2 - n_{\text{clad}}^2) d)/\lambda \)
Fig. 27 Cross-section of gain guided stripe geometry 980 nm single-mode laser.

Fig. 28 Cross-section of an index guided ridge waveguide 980 nm single-mode laser.

Fig. 29. Schematic cross-section of a ridge waveguide laser.