**ECE 4211 QUIZ2 Design Pass2 Take HomeV2** 042017due 04202017 F. Jain**. NAME____**Sol

Review class on Monday April 17th in LH 205 at 6:00pm

First pass laser & solar designs 16 points each. Second pass 4 points each for solar and laser.

**Q.1 Design** a double heterostructure quantum dot laser operating at 1.32 μm and emitting 1. 5mW using InGaAsP-InP system. The device parameters including Z(T), $\Delta V_s$, $\eta_q$ are the same as in Design Part #1. Fig. 1 shows the cross section with three layers of dots and 4 barrier layers.

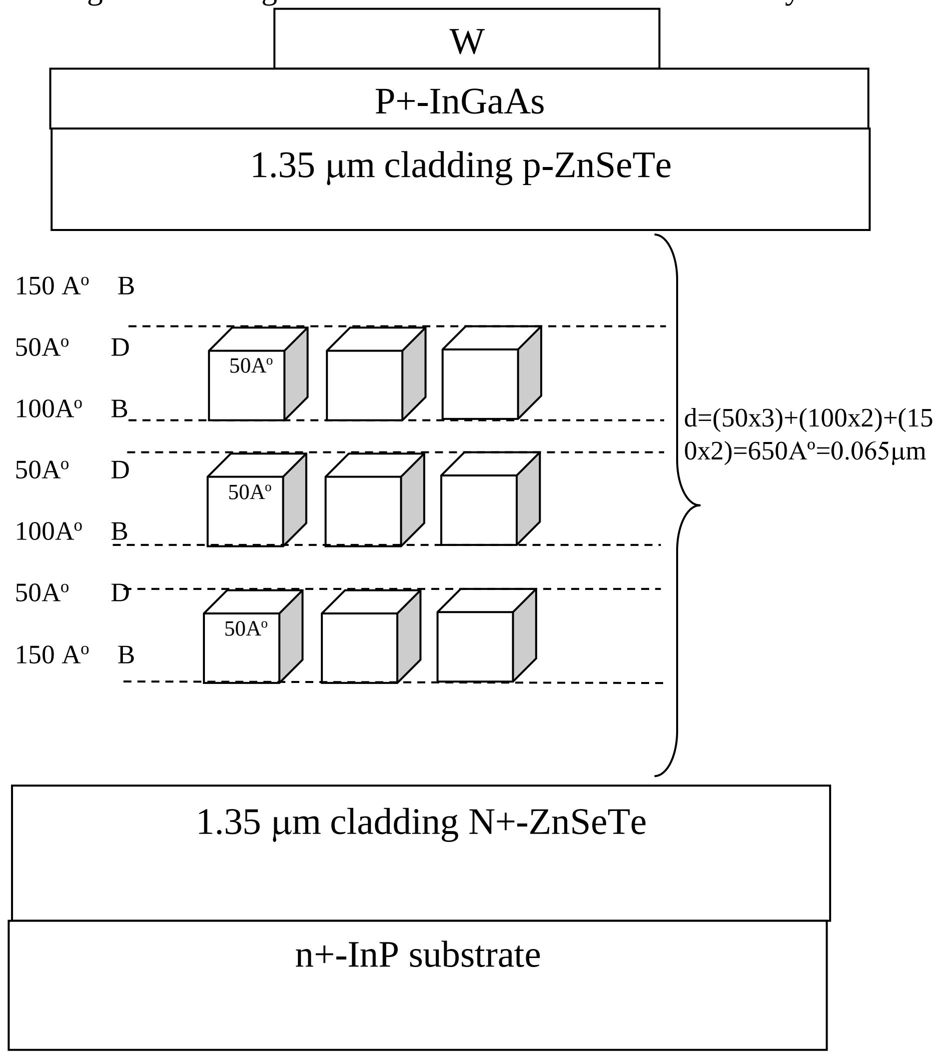



Fig. 1 Double heterostructure InGaAsP-InP quantum dot laser with ZnSeTe cladding layers. The active layer comprises three quantum dot (50Å each) and four barrier layers with active layer thickness d= 650Å = 0.065μm.

Quantum dot energy levels are are shown below.

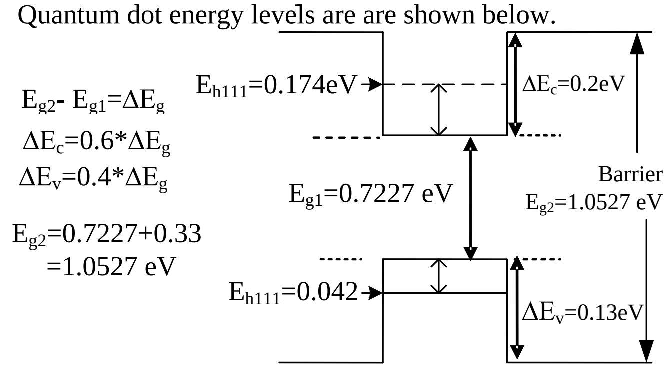



**Fig. 2** Energy levels in quantum dot $E_{g1}$ having barrier layers $E_{g2}$ on either side.

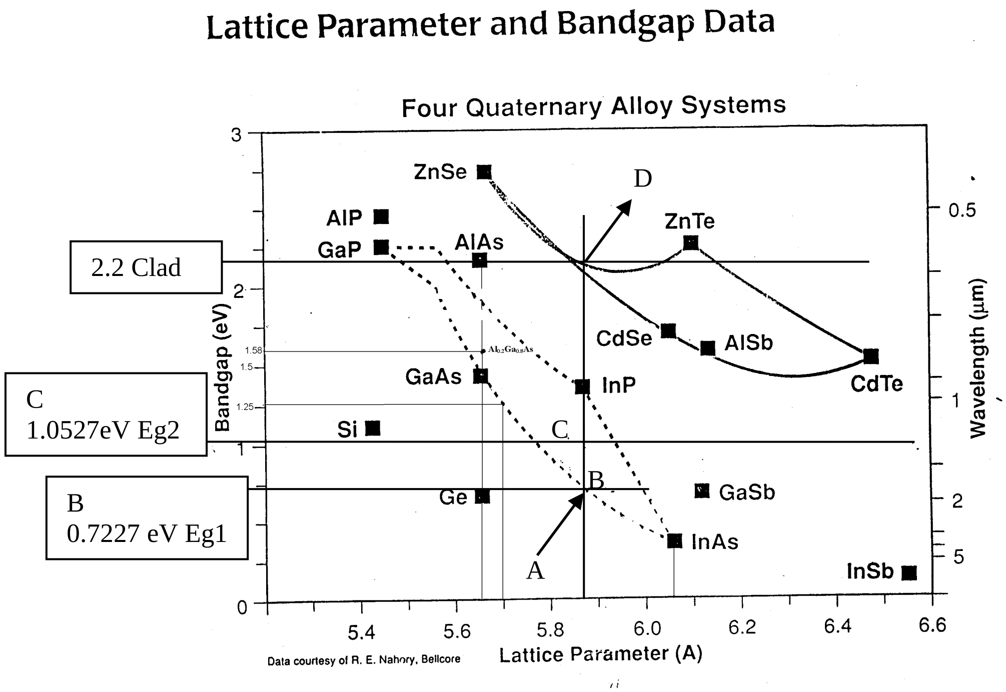



Fig. 3 Energy gap as a function of lattice parameter.

**Point A** is 0.7eV (intersection of dashed line from GaAs to InAs and vertical line from InP) composition In0.53Ga0.47As,
**Point B** is quantum dot at 0.7227eV (intersection of solid horizontal line at 0.7227eV and vertical line from InP). Composition-you need to find.
**Point C** is barrier layer at 1.0527eV (intersection of solid horizontal line at 1.0.527eV and vertical line from InP). Composition-you need to find.
Point D is cladding layer at the intersection of 2.2eV horizontal line and vertical line via InP.

Q.1 (a) **Find the composition of quantum dot** in the active layer using Fig. 3.

$E_g$ for **dots:** $h\nu = 1.24/1.32\mu m = 0.9393 = E_{g1} + E_{e111} + E_{hh111} + 0.0006$ (in eV).
Fig. 2 shows the electron energy level $E_{e111}$ and hole energy level $E_{hh111}$ of Quantum Dots (size 50x50x50Å). The dots are sandwiched between barriers with energy gap $E_{g2}$. The differences $\Delta E_g = (E_{g2} - E_{g1})$ =0.33eV is distributed in conduction band and valence band as $\Delta E_c$ = 0.2 and $\Delta E_v$=0.13eV. $\Delta E_c$ = 0.2eV is greater than $E_{e111}$(=0.174eV), and $\Delta E_v$=0.13eV is greater than $E_{hh111}$ (= 0.042eV). Thus, both electons and holes are confined in the quantum dot by the barrier layer.
This gives **quantum dot energy gap $E_{g1}$**
**$E_{g1}$**=0.9393 – ($E_{e1}$ + $E_{hh1}$ + 0.0006) = 0.9393 – (0.174 + 0.042 + 0.0006) = 0.7227eV.

In Hint set 2 we provided Indium composition to be:
0.53 + [(0.7227 -0.7)/(1.34-0.7)]*0.47= 0.53 + 0.03546*0.47=0.5466
We also provided phosphorus composition to be= [(0.7227 -0.7)/(1.34-0.7)]*1.0 =0.0354
Gallium is 1.0-Indium = 1- 0.5466 = 0.4534.
Arsenic is = 1-phosphorus = 1.0 – 0.0354= 0.9646

So the composition is $In_{0.5466}$ $Ga_{0.4534}$ $As_{0.9646}P_{0.0354}$

The index of refraction is found using refraction equation for $In_xGa_{1-x}As_yP_{1-y}$ given below

$x$=0.5466; 1-$x$=0.4534; $y$=0.9646; 1-$y$=0.0354

$$n(x,y) = 3.52xy + 3.39x(1-y) + 3.60y(1-x) + 3.56(1-x)(1-y).$$

=3.52*0.5466*0.9646+3.39*0.5466*0.0354+3.60*0.9646*0.4534+3.56*0.4534*0.0354=3.5531

**The index of refraction $n_r$ of quantum dot is 3.5531.**

Q.1 (b) **Find the composition of barrier layer** which has a higher band gap ($E_{g1}$) such that $\Delta E_c + \Delta E_v$ are larger than the electron $E_{e111}$ and hole $E_{hh111}$ levels.

Since $\Delta E_g$ =0.33 eV confines elections and holes, the barrier layer energy gap $E_{g2}$ is:

$\mathbf{E_{g2} = E_{g1} + \Delta E_g = 0.7227 + 0.33 = 1.0527}$ **eV**

Indium composition of quantum well barrier point C is:

0.53 + [(1.0527 -0.7)/(1.34-0.7)]*0.47= 0.53 + 0.5510*0.47=0.7889

We also provided phosphorus composition to be= [(1.0527 -0.7)/(1.34-0.7)]*1.0 =0.5510

Gallium is 1.0-Indium = 1- 0.7889 = 0.2111.

Arsenic is = 1-phosphorus = 1.0 – 0.5510= 0.449

So the composition is $In_{0.7889}\ Ga_{0.2111}\ As_{0.449}P_{0.551}$

The index of refraction is found using refraction equation for $In_xGa_{1-x}As_yP_{1-y}$ given below

$x$=0.7889; 1-$x$=0.2111; $y$=0.449; 1-$y$=0.551

$$n(x,y) = 3.52xy + 3.39x(1-y) + 3.60y(1-x) + 3.56(1-x)(1-y).$$

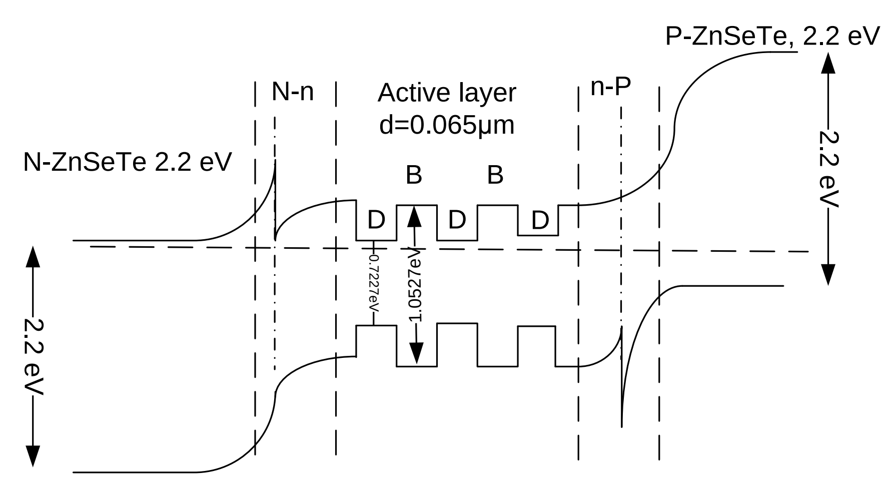
= 3.52*0.7889*0.449+3.39*0.7889*0.551+3.60*0.449*0.2111+3.56*0.2111*0.551= 3.4757.

**The index $n_r$ of barrier layer is 3.4757.**

Q.1 (c) Draw the energy band diagram given the cladding energy band gap Eg3 = 2.2eV.

Assume $\Delta E_c = 0.6*(E_{g3} - E_{g2}) = 0.6*(2.2- 1.0527) = 0.6*1.1473=0.6883$eV, and

$\Delta E_v = 0.4589$eV. HINT: See the Quiz1 Bonus question Solution energy band plot.




Q.1 (d) Find the confinement factor Γand threshold current density $J_{th}$

The cladding band gap $E_{g3}$= 2.2eV. Index is $n_{clad} = \sqrt{8.5} = 2.915$, $E_g = 2.20eV$,
The index of quantum dot active layer $n_{rQD}$ = ($n_r$ of quantum dot + $n_r$ of barrier)/2

= (3.5531+3.4757)/2 = 3.5144

(d1) Find the confinement factor Γ using $\Gamma = \frac{V^2}{2+V^2}$, where $V^2 = \frac{4\pi(d^2)(n_{active}^2 - n_{cladding}^2)}{\lambda^2}$.

$$V^2 = \frac{4*3.14*(0.065\mu)^2 *(3.5144^2 - 2.915^2)}{(1.32\mu)^2} = 0.1173$$

Confinement factor $\Gamma = \frac{V^2}{2+V^2} = \frac{0.1173}{2+0.1173}$ = 0.1173/2.1173 = 0.0554

⇒d2) Find threshold current density and show that it is under **200 A/cm²**; explain if it is not.
**Solution: Threshold current density:**

$$J_{th} = \frac{8\pi n_r^2 q d \Delta\nu_s}{\Gamma\eta z(T)}\left(\frac{1}{\lambda^2}\right)(\alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right))$$

α=20cm$^{-1}$; Γ = 6.832*10$^{-3}$; $\eta_q$=0.9; $\Delta\nu_s$=1.2*10$^{12}$; z(T)=0.5; $R_1 = R_2$ = 0.3, d = 0.065 micron.

We take cavity length L=500µm;

$$J_{th} = \frac{8\pi * 3.5144^2 * 1.6*10^{-19} * 0.065*10^{-4} * 1.2*10^{12}}{0.0554*0.9*0.5}\left(\frac{1}{(1.32*10^{-4})^2}\right)\left[20 + \frac{1}{2*500*10^{-4}} * \ln\left(\frac{1}{0.3*0.3}\right)\right]$$

$$= \frac{0.38739*10^{-9}}{0.02493}\left(\frac{1}{(1.32*10^{-4})^2}\right)\left[20 + \frac{1}{0.1} * \ln\left(\frac{1}{0.09}\right)\right]$$

$$= 0.891*(20 + 24.079) = 39.27 \text{ A/cm}^2$$

**Assume**: $\frac{dn_r}{d\lambda}$ =1.5 µm$^{-1}$, Spontaneous line width ***$\Delta\nu_s$ = 1.2x10$^{12}$ Hz and Z (T) ≈0.5***

Given active layer doping is 10$^{16}$ cm$^{-3}$ as shown in Q.1..
Internal Quantum efficiency = $\eta_q$ = 0.9
Absorption coefficient $\alpha = \alpha_{Diffraction} + \alpha_{Free\ carrier} + \alpha_{scattering}$ =20 cm$^{-1}$
Electron effective masses $m_n = 0.067m_o$, heavy hole mass $m_p = 0.62m_o$,
**Determine** the end reflectivity $R_1$, $R_2$. (If you cannot find them, use $R_1=R_2$=0.3)

Minority hole lifetime $\tau_p$ =5x10$^{-9}$ sec
Hole diffusion coefficient $D_p$=10 cm$^2$/s
Minority electron lifetime $\tau_n$ =1x10$^{-8}$ sec
Electron diffusion coefficient $D_n$=50 cm$^2$/s

Q2. **Second pass Solar Cell Design.** One of the questions in Pass One we asked was:
How would you improve the efficiency of your current cell?
The solution provided in solution set 10 includes many methods listed below.
The efficiency is given by $\eta = (V_m*I_m)/P_{in} = [(V_m*I_m)/ V_{oc}*I_{SC}\ ] *[(V_{oc}*I_{SC})/P_{in}]$
$\eta$ = Fill Factor * $[(V_{oc}*I_{SC})/P_{in}]$.
The efficiency can be increased by increasing fill factor, $V_{oc}$ and Isc.
$V_{oc}$ can be increased by reducing reverse saturation current.
$I_{SC}$ can be improved by removing defects in the absorber layer where photogenerated electron hole pairs can recombine before separation across the p-n junction.
$V_{oc}$ can be increased by tandem cells. But tandem cells reduce $I_{SC}$ so a tradeoff is there.
Isc can be improved by having a lower energy gap absorber. But lower energy gap reduced $V_{oc}$, so there is a tradeoff.
Having a heterojunction cell whose window region is of larger energy gap reduces loss in window region.
Use of n+-n/p-p+ cell improves $V_{oc}$ in both homojunction as well as heterojunction cells.

**Q2(a) Do you think increasing the doping** of p-Si from $8*10^{16}$ to $8*10^{17}$ $cm^{-3}$ will improve the conversion efficiency.
If yes, determine the new values of open circuit voltage, maximum power point $P_m = V_m I_m$ and the fill factor (FF). All other parameters are the same.

**Parameters:**
Design an n+-p Si solar cell for air mass m=1 (AM1). Assume that the incident radiation for AM1 (Fig. 1) is 92.5mW/$cm^2$. $I_L=I_{SC}$ = 38.97mA/$cm^2$.
Follow cell design example of **Section 6.12, page 511-518.**
**Given:** p-Si crystalline wafer with doping of **$8x10^{16}cm^{-3}$**.
**$n^+$-side**: Donor concentration $N_D=10^{20}$ $cm^{-3}$, minority hole lifetime $\tau_p=2x10^{-6}$ sec.
Minority hole diffusion coefficient $D_p$=12.5 $cm^2$/sec.
**p-side**: Acceptor concentration $N_A$= **$8x10^{17}cm^{-3}$**, $\tau_n=10^{-5}$sec. $D_n$=40 $cm^2$/sec.
Junction area A=1 $cm^{-2}$, $n_i$ (at 300K) = $1.5x10^{10}cm^{-3}$, $\varepsilon_r$ (Si)=11.8, $\varepsilon_0=8.85x10^{-14}$ F/cm, $\varepsilon=\varepsilon_0\varepsilon_r$. Assume all donors and acceptors to be ionized at T=300°K.

**Q2(a) Do you think increasing the doping** of p-Si from $8*10^{16}$ to $8*10^{17}$ $cm^{-3}$ will improve the conversion efficiency.

**Solution: yes, it increases $V_{oc}$, $V_m$, $I_m$ and FF and Voltage factor.**
We need to find $V_{OC}$ to find $V_m$, and to find $V_{OC}$ we need to find the reverse saturation current $I_s$ as $I_{SC}$ is given to be 38.97 mA. First, we determine the open circuit voltage. This will be followed by maximum power point $P_m = V_m I_m$ and the fill factor (FF) and voltage factor.

Here, $V_{OC} = \frac{kT}{q}\ln\left(\frac{I_{SC}+I_S}{I_S}\right)$. And $I_s$ is expressed as $I_s = q*A*\left[\frac{D_p P_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right]$

**p-side:** $n_{po} = \frac{n_i^2}{N_A} = \frac{(1.5*10^{10})^2}{8*10^{17}} = 2.8125x10^2\, cm^{-3}$, $L_n = \sqrt{D_n\tau_n} = \sqrt{40*10^{-5}} = 2*10^{-2}cm$

**$n^+$-side: (window region)**

$p_{no} = \frac{n_i^2}{N_D} = \frac{(1.5*10^{10})^2}{10^{20}} = 2.25cm^{-3}$ and $L_p = \sqrt{D_p\tau_p} = \sqrt{2*10^{-6}*12.5} = 5*10^{-3}\,cm$

$$I_s = qA\left[\frac{D_n n_{po}}{L_n} + \frac{D_p P_{no}}{L_p}\right]$$

$$I_S = 1.6*10^{-19}*1*\left[\frac{40*2.8125*10^2}{2*10^{-2}} + \frac{12.5*2.25}{5*10^{-3}}\right]$$

$$= 1.6*10^{-19}*1*\left[\frac{40*2.8125*10^2}{2*10^{-2}} + 2.25*2.25*10^3\right]$$ $=0.9*10^{-13} + 0.9*10^{-15} = 0.9*10^{-13}$**,** $I_s = 0.9*10^{-13}$ A

Substituting $I_{SC}$ and $I_s$ in the $V_{OC} = \frac{kT}{q}\ln\left(\frac{I_{SC}+I_S}{I_S}\right)$, we can neglect Is in the numerator

$V_{oc} = 0.0259 \ln[(38.97x10^{-3} + 0.9x10^{-13})/ 0.9x10^{-13}] = 0.0259*26.794 = 0.6939$ V, $V_{OC} = 0.6939$ V

## Determine the maximum power point $V_m$, $I_m$

$V_m$ and Im are expressed as: .

$$V_m = V_{OC} - \frac{kT}{q}\left[\ln\left(1+\frac{qV_m}{kT}\right)\right] \quad \text{and } I_m = I_S\left(e^{\frac{qV_m}{kT}} - 1\right) - I_{SC}$$

The maximum power point $V_m$ and $I_m$

$$V_m = V_{OC} - \frac{kT}{q}\left[\ln\left(1+\frac{qV_m}{kT}\right)\right] \quad \text{and } I_m = I_S\left(e^{\frac{qV_m}{kT}} - 1\right) - I_{SC}$$

Substitute $V_{OC}$ and kT/q = 0.0259V in $V_m = V_{OC} - \frac{kT}{q}\left[\ln\left(1+\frac{qV_m}{kT}\right)\right]$

$V_m = 0.6939 - 0.0259\left[\ln\left(1+\frac{V_m}{0.0259}\right)\right]$. Since $V_m$ is on both sides, we need to write a short program or do trial and error substitution. We know $V_m$ is less than $V_{OC}$, so we guess it to be 0.5V. for this guess we tabulate Left Hand Side (LHS) and Right Hand Side (RHS) until the value of $V_m$ makes both sides equal.

| LHS | RHS |
|---|---|
| 0.5V (first guess) | 0.6159 |
| 0.615 | 0.6107 |
| 0.611 | 0.6109 |

So $V_m = 0.610$ V and $I_m$ is obtained by substituting $V_m$ in the current equation.

$I_m = 0.9*10^{-13}[\exp(0.610/0.0259) - 1] - 38.97X10^{-3}$

$= 1.523x10^{-3} - 38.97x10^{-3} = -37.447$ mA

Maximum power $P_m = V_m*I_m = 37.447*0.610 = 22.8426$ mW (for a cell of area 1 $cm^2$).

Fill Factor FF = $\frac{V_m I_m}{V_{OC} I_{SC}} = \frac{0.610*37.447mA}{0.6939*38.97mA}$ =0.8447

**FF = 0.8447**

**Voltage Factor : $V_{OC}/(E_g/q) = 0.6939/1.1 = 0.6308$**

**Compute dominant losses:**

(a)Long wavelength photons that are not absorbed. ***Same as design Pass 1***

These are photons below the energy gap $E_g$ = 1.1eV for Si. These are shown in the above Figure, in area of region 1, 2 & 3, and a triangle F'/1.1 (D) /1.16 (D') micron point on x-axis.

The solar power in these regions is: Region #1=68.4 $W/m^2$, Region =2 72.69 $W/m^2$, and Region#3 = 35.77 $W/m^2$.These values have been calculated in solar design before.

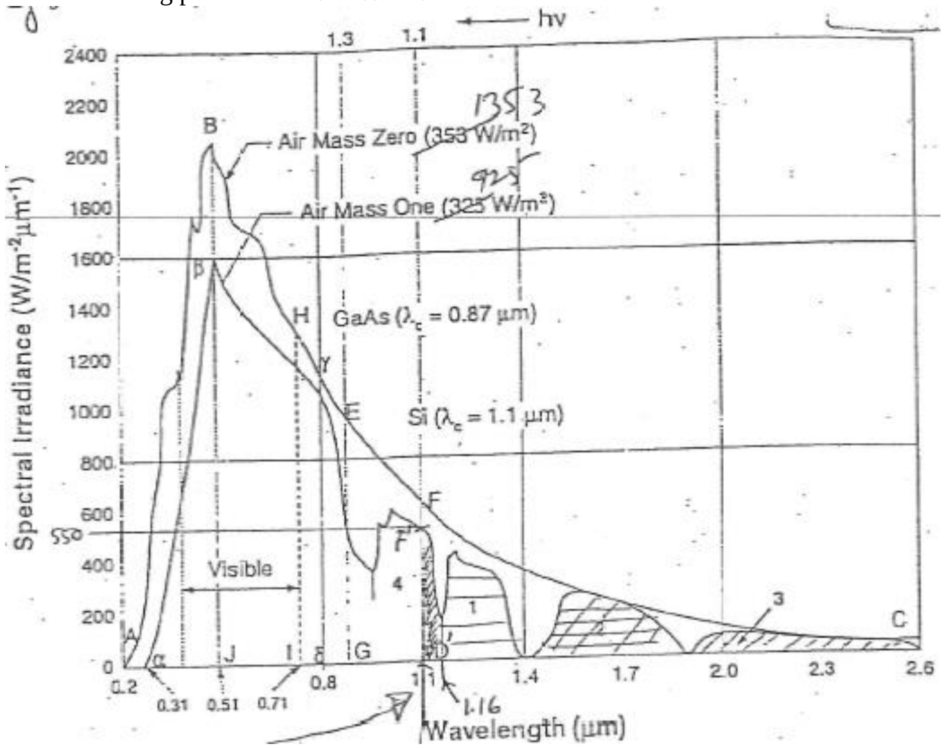
Region #F'DD' (in hint set triangle F'DK) power is 16.5 $W/m^2$, as shown below.

$$Area = \frac{1}{2}*DF'*DD' = \frac{1}{2}*550*(1.16-1.1)\mu = \frac{1}{2}*550*0.06 = 16.5W/m^2$$

**Total long wavelength photon loss is 193.36 $W/m^2$** or 19.36 $mW/cm^2$.

The % of long wavelength light loss at AM1 in Si = $\frac{193.36}{925} = 20.9\%$.

The remaining power is 731.64 W/m$^2$.




Two curves show solar spectral irradiance for AM1 and AM0.

(b) Excess Energy Loss (energy above $E_g$ =1.1eV not used to create electron-hole pairs). **Same as Design pass 1**

*Excess photon energy not utilized in electron hole pair generation* $= h\upsilon - E_{g,Si}$

Let AM1 plot above $\lambda < 1.1\mu m$ or $h\upsilon > 1.1 eV$ is divided in several regions. The exact way is to find the area under the curve numerically. Regions are: Triangle $\alpha\beta J$, Trapezoid $J\beta\gamma\delta$, Trapezoid marked as #4. These are calculated next.

Excess photon energy lost in spectral region represented by triangle $\alpha\beta J$

$$\Delta\alpha\beta J \Rightarrow \text{Photon energy at } \alpha = \frac{1.24}{0.31} = 4eV \text{ and Photon energy at } J = \frac{1.24}{0.51} = 2.43eV$$

$$h\upsilon_{ave} = \frac{4+2.43}{2} = 3.12eV,$$

$$\text{Excess photon energy} = 3.21 - 1.1 = 2.11eV$$

Area if the $\Delta\alpha\beta J = \beta J * \frac{\alpha J}{2} = 1550 * \frac{(0.51-0.31)}{2} = 155 W/m^2$

Excess energy not used $= \frac{155}{3.21} * 2.11 = 101 W/m^2$

$$\text{Trapezoid} j\beta\gamma\delta \Rightarrow h\upsilon \text{ at } \beta J \Rightarrow \frac{1.24}{0.51} = 2.43eV, h\upsilon \text{ at } \beta J \Rightarrow \frac{1.24}{0.51} = 2.43eV$$

$$h\upsilon_{ave} \Rightarrow \frac{1.476+2.43}{2} = 1.953eV$$

$$\text{Trapezoid} = \text{Rectangle } \gamma\gamma J\delta + \Delta\gamma\gamma\beta = 950*(0.84-0.51)+(0.84-0.51)*(1550-950)/2 = 412.5W/m^2$$

Excess energy lost $=(412.5/1.953)*0.853 = 181.16W/m^2$

Trapezoid #4, Rectangle #5 & Rectangle #6 can be combined by a rectangle

$$=-500*(0.84-1.1)=130W/m^2$$

$$h\upsilon_{ave} = \frac{\left(\frac{1.24}{0.84}+\frac{1.24}{1.1}\right)}{2} = \frac{1.476+1.127}{2} = 1.3eV$$

Excess energy = $h\upsilon_{ave} - E_g = 1.3-1.1 = 0.2eV$

Excess energy not used = $\frac{130}{1.3eV}*0.19 = 19W/m^2$

**Total excess energy loss** = 101 + 180.16 + 19 = 300.16 W/m$^2$.

The percentage loss is 300.16/925 = 32.4%.

**Available power is 731.64 – 300.16 = 431.48 W/m$^2$.**

(c) **Voltage factor** is defined as the ratio of $V_{OC}$ and $E_g/q$. Voltage factor $= \frac{V_{OC}}{E_g/q}$=0.6939/1.1 = 0.6308.

The loss is 431.48 x (1-0.6308) = 159.30 W/m2.

The % loss is 159.30/925 = 17.22%.

This leaves available power of (431.48 – 159.30) = 272.18 W/m$^2$.

Remaining % efficiency is 46.66-17.22 = 29.44%

(d) **Fill Factor (FF)** is defined as $(V_m I_m)/(V_{OC} I_{SC})$. $\frac{V_m I_m}{V_{OC} I_{SC}} = \frac{0.610*37.447mA}{0.6939*38.97mA}$ = FF = 0.8447

The power loss due to FF is (1-0.8447)*272.18=42.269 W/m$^2$.

The % loss is 42.269/925 = 0.0456=4.56%.

The remaining power is (272.18 – 42.269) = 229.911 W/m$^2$.

**The remaining cell efficiency is 29.44- 4.56= 24.88 %.**

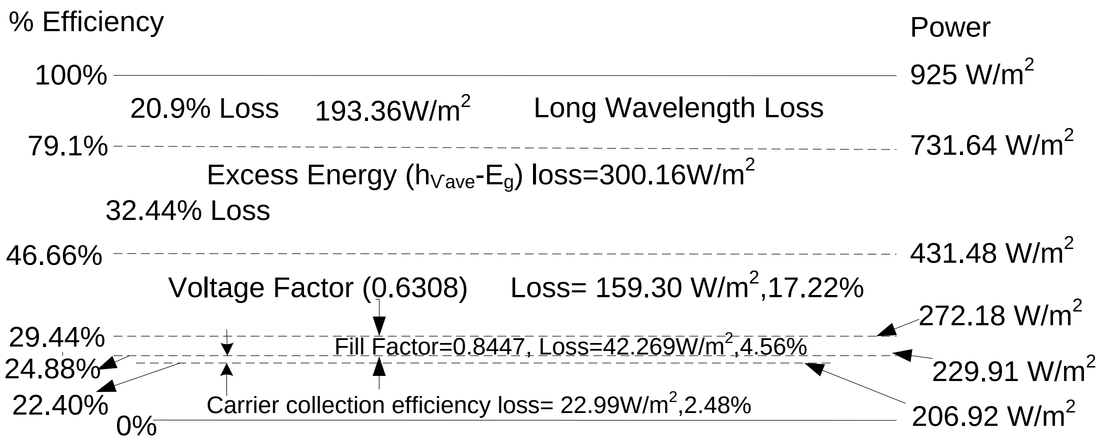
OPTIONAL: **Collection efficiency** of photo-generated electrons and holes. In crystalline Si this loss may be 10%. However, it is significant in poly-crystalline Si substrates, where it is ~20%.

A 10% loss reduces the available power to (229.91- 22.99) = 206.92 W/m$^2$. The % loss in efficiency is 22.99/925 = 2.48%.

**The remaining cell efficiency is 24.88- 2.48 = 22.40%.**

**Series resistance loss**: Series resistance loss $R_s$ and other losses.

**Q2(b) Bar chart;**




Bar Chart of losses in Si solar ce