UConn ENGR-ECE 4243/6243_Solution Set 4B   09272016 F. Jain

Photon absorption and solar cells, photon emission and LEDs,

Q1 Figure 1 shows two 10.0 μm thick Si and GaAs samples illuminated by a 10 mW photon source. Assume the source to be monochromatic and emitting photons with energy hν=1.9 eV.

(a) Given the absorption coefficient α(hν=1.9eV of λ =0.65 μm) = 3000 cm^{-1} in crystalline Si and index of refraction $n_r = \sqrt{11.8} \approx 3.45$, find the power absorbed in Si in one pass.

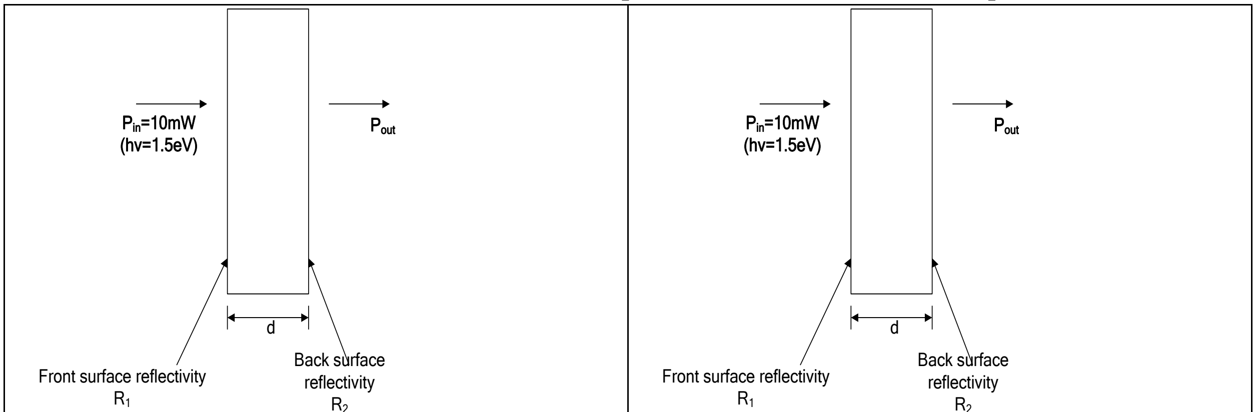


Fig. 1a Photon absorption in Si.	Fig. 1b Photon absorption in GaAs.

(b) Find the power absorbed in GaAs. Use the plot of absorption coefficient in Notes (page 355) for GaAs at 1.9eV. For index of refraction of GaAs, use home work on laser diode where AlGaAs-GaAs info is provided.

(c) If the crystalline Si is replaced by amorphous Si (a-Si), find the absorbed power in film of 1 μm in thickness. Given α (hν=1.9eV of λ =0.65 μm) = 10,000 cm^{-1} in a-Si sample. Assume the same index of refraction.

(d) Find out the portion of absorbed energy that is not used up in electron-hole pair (EHP) generation either in crystalline and GaAs.

Hint: The excess energy is ($h\nu - E_g$) per photon. It is wasted in the form of heat. Multiplying this by the number of photons will give the power that is wasted.

Solution

Problem (a)

$R_1 = R_2 = \left(\frac{n_r - n_{air}}{n_r + n_{air}}\right)^2$

$n_r = \sqrt{11.8} = 3.4351$

$R_1 = R_2 = \left(\frac{3.4351-1}{3.4351+1}\right)^2 = 0.30146$

Power inside the Si wafer = $P_{in}'(X=0) = P_{in} - P_{in} * R_1$

$= P_{in} (1 - R_1)$

$= P_{in} (1-0.30146)$

= 10mW * 0.699= 6.99mW

Power reading at X=d = $P_{out} = P_{in}$*[exp (-αd)]

= 6.99mW*exp (-3000*10*10^{-4})

= 0.348mW

Power absorbed = $P_{in}' - P_{out}'$ = Pabs1 = 6.9854 – 0.3478 = 6.6376 mW ≈ 6.638mW

Optional:
Some of the power that reaches x=d is reflected back and its value is
$P_{out}'' = P_{out}' * R_2 = 0.348$ mW * 0.301= 0.105 mW

Of this, the fraction absorbed (in travel to surface R_1= $P_{abs2} = P_{out}*(1-e^{-\alpha d})$
$= 0.105mW\ (1-e^{-3})$
$= 9.9772x10^{-2}$ mW=0.099772mW

P_{out}'' will be reflected from surface #1 (R_1P_{out}'') and some of this will be absorbed. But these magnitudes are smaller.

Total power absorbed = $P_{abs} = P_{abs1} + P_{abs2}$
= 6.637623 + 0.099772 = 6.737395 mW ≈ 6.737 mW

For simplicity, this problem will assume that P_{abs1} = 6.638mW for simplicity.

Problem (b)
Crystalline Si is replaced with amorphous Si in part (a)
So, $R_1 = R_2 = 0.303$
Power inside the Si wafer = $P_{in}'(X=0) = P_{in} - P_{in} * R_1$
$= P_{in}\ (1 - R_1)$
$= P_{in}\ (1-0.301)$
= 10mW * 0.699= 6.99mW

Power reading at X=d = $P_{out}' = P_{in}'[\exp(-\alpha d)]$
$= 6.99mW*\exp(-10000*1*10^{-4})$
= 2.570mW

Power absorbed = $P_{in}' - P_{out}'$ = Pabs1 = 6.9854 – 2.5698 = 4.4156mW ≈ 4.416 mW.

Optional:
Some of the power that reaches x=d is reflected back and its value is
$P_{out}'' = P_{out}' * R_2 =$ 2.570 mW * 0.301= 0.774 mW

Of this, the fraction absorbed (in travel to surface R_1) = $P_{abs2} = P_{out}''(1-e^{-\alpha d})$
$= 0.774mW*(1-e^{-1})$
= 0.489 mW.

P_{out}'' will be reflected from surface #1 (R_1P_{out}'') and some of this will be absorbed. But these magnitudes are smaller.

Total power absorbed = $P_{abs} = P_{abs1} + P_{abs2}$
= 4.416 + 0.489 = 4.905 mW.

For simplicity, this problem will assume that P_{abs1} = 4.416 mW for simplicity.

Problem (c)

n_r for GaAs = $\sqrt{13.18 \square - 3.12\xi}$ (From the Q2, HW set 6)= $\sqrt{13.18}$ (For GaAs ξ =0)=3.63

$R_1 = R_2 = \left(\frac{n_r - n_{air}}{n_r + n_{air}}\right)^2$

$R_1 = R_2 = \left(\frac{3.63-1}{3.63+1}\right)^2 = 0.323$

Power inside the Si wafer = P_{in}'(X=0) = P_{in} - P_{in} * R_1
= P_{in} (1 - R_1)
= P_{in} (1-0.323)
= 10mW * 0.677 = 6.77mW

From α Vs E_g plot, the α(hν) for GaAs = $3*10^4$ cm^{-1}

Power reading at X=d = P_{out}' = P_{in}'[exp (-αd)]
= 6.77mW*exp (-30000*10*10^{-4})
= 6.77mW*e^{-30}
= $6.335*10^{-13}$W

Power absorbed = P_{in}' – P_{out}' = Pabs1 = 6.77 – $6.335*10^{-13}$= 6.77 mW.

Problem (d)

c-Si 1.9-1.1=0.8eV
a-Si 1.9-1.55=0.35eV
GaAs 1.9-1.424=0.476eV

Problem (e)

Portion of photon energy not used up in EHP generation

For Crystalline Si:
Photon energy required to generate an EHP = 1.1 eV in Si
Excess energy per photon = 1.9eV – 1.1eV = 0.8eV.

Number of Photons absorbed / sec = $\frac{P_{abs1}}{h\vartheta} = \frac{6.623*10^{-3}}{1.9*1.6*10^{-19}} = 2.18*10^{16}\ photons/\sec$

Excess energy not used / sec = 2.18 * 10^{16} * 0.8 * 1.6 * 10^{-19} = 2.7904 mW

For GaAs :
Photon energy required to generate an EHP = 1.424 eV in Si
Excess energy per photon = 1.9eV – 1.424eV = 0.476eV.

Number of Photons absorbed / sec = $\frac{P_{abs1}}{h\vartheta} = \frac{6.77*10^{-3}}{1.9*1.6*10^{-19}} = 2.23*10^{16}\ photons/\sec$

Excess energy not used / sec = 2.23 * 10^{16} * 0.476 * 1.6 * 10^{-19} = 1.698 mW

Q2. Fig. 2 shows an n^+-P Si diode with following device / material parameters.

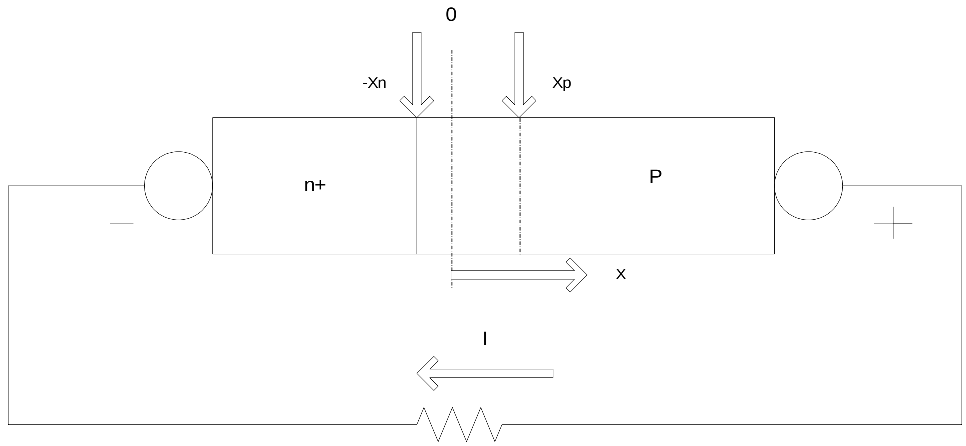


Fig. 2. An abrupt n^+-p Si solar cell (the light is from the left or n^+ side)

Given: **n^+-side:** Donor concentration $N_D = 10^{20}$ cm^{-3}, minority hole lifetime τ_p=2x10^{-6} sec. Minority hole diffusion coefficient D_p=12.5cm^2/sec.
p-side: Acceptor concentration N_A=5x10^{17} cm^{-3}, τ_n=10^{-5} sec. D_n=40cm^2/sec.
Junction Area = A = 1 cm^{-2}, n_i (at 300K)=1.5x10^{10}cm^{-3}, ε_r (Si)= 11.8, ε_0=8.85x10^{-14}F/cm, $\varepsilon=\varepsilon_r\varepsilon_0$. Effective mass: electrons $m_e=m_n=0.26m_o$, holes $m_h=m_p$= 0.64 m_o,
Assume all donors and acceptors to be ionized at T=300K.

(a) Determine the open circuit voltage Voc, if the light generated current I_L = 27mA. Assume $I_L=I_{SC}$ (the short circuit current). ***Find I_s value like HW#2 solution set. This is n-p diode.***
(b) Determine the maximum output power P_{mp}.
(c) Find the fill factor FF.
(d) What is the effect on V_{oc} of raising the operating temperature from 300K to 500K.

Solution
Problem (a)

$$V_{oc} = \frac{kT}{q}\ln\left(\frac{I_s + I_{sc}}{I_s}\right)$$

Reverse saturation current = $I_s = \frac{qAD_n n_{p0}}{L_n} + \frac{qAD_p p_{n0}}{L_p}$

$n_{p0} = \frac{n_i^2}{N_A} = \frac{2.25*10^{20}}{5\times10^{17}} = 4.5*10^2 cm^{-3}$

$p_{n0} = \frac{n_i^2}{N_D} = 2.25 cm^{-3}$

$L_n = \sqrt{D_n\tau_n} = \sqrt{40*10^{-5}} = 2*10^{-2} cm$

$L_p = \sqrt{D_p\tau_p} = \sqrt{12.5*2*10^{-6}} = 5*10^{-3} cm$

$I_s = 1.6*10^{-19}*1*\left[\frac{40*4.5*10^2}{2*10^{-2}}+\frac{12.5*2.25}{5*10^{-3}}\right]$, neglect the second term as it is smaller than first.

I_s = 0.145 pA.

Open circuit voltage at 300K,

$$V_{oc} = 0.0259\ln\left[\frac{0.145*10^{-12}+27*10^{-3}}{0.145*10^{-12}}\right] = 0.6721 Volts$$

Problem (b)

The maximum power output $P_m = V_m I_m$ depends on the V_m and I_m values. Here, the maximum power output occurs when V_m is expressed as: $V_m = V_{oc} - \frac{kT}{q}\ln\left[1+\frac{qV_m}{kT}\right]$.

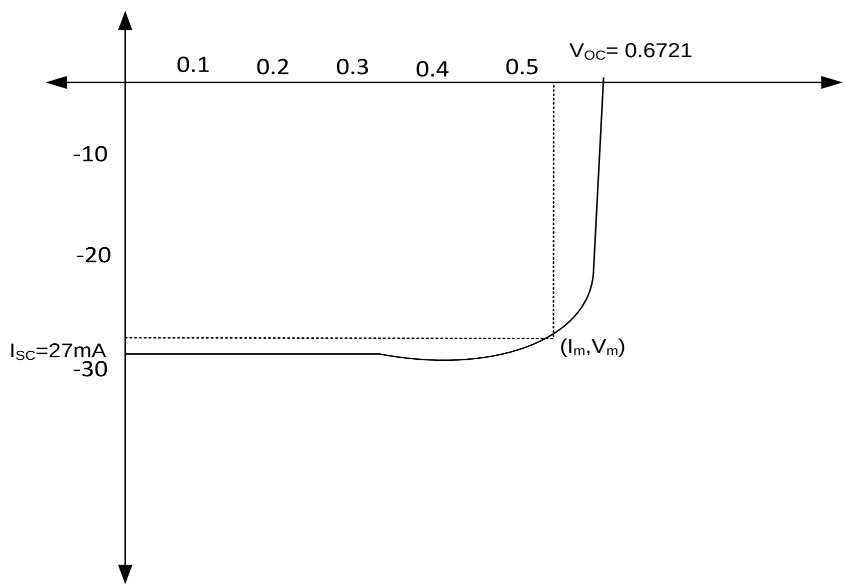


Fig. P4. I_m-V_m point shown on the solar characteristics (curve is not joined smoothly).

Substituting for V_{oc}, we get,

$$V_m = 0.6721 - 0.0259\ln\left[1+\frac{qV_m}{kT}\right] \qquad \text{(A)}$$

Find V_m: Write a program or do by trial & error.

V_m = 0.59V makes both LHS & RHS almost the same.

Get I_m by substituting in V_m in I-V equation.

$$I_m = I_s\left(e^{\frac{qV_m}{kT}} - 1\right) - I_{SC}$$

$$= 0.145*10^{-12}\left(e^{\frac{0.59}{0.0259}} - 1\right) - 27*10^{-3} = 1.13\times10^{-13} - 27\times10^{-3} = -25.87 mA$$

$$P_m = |25.87 mA| * 0.59 = 15.26 mW$$

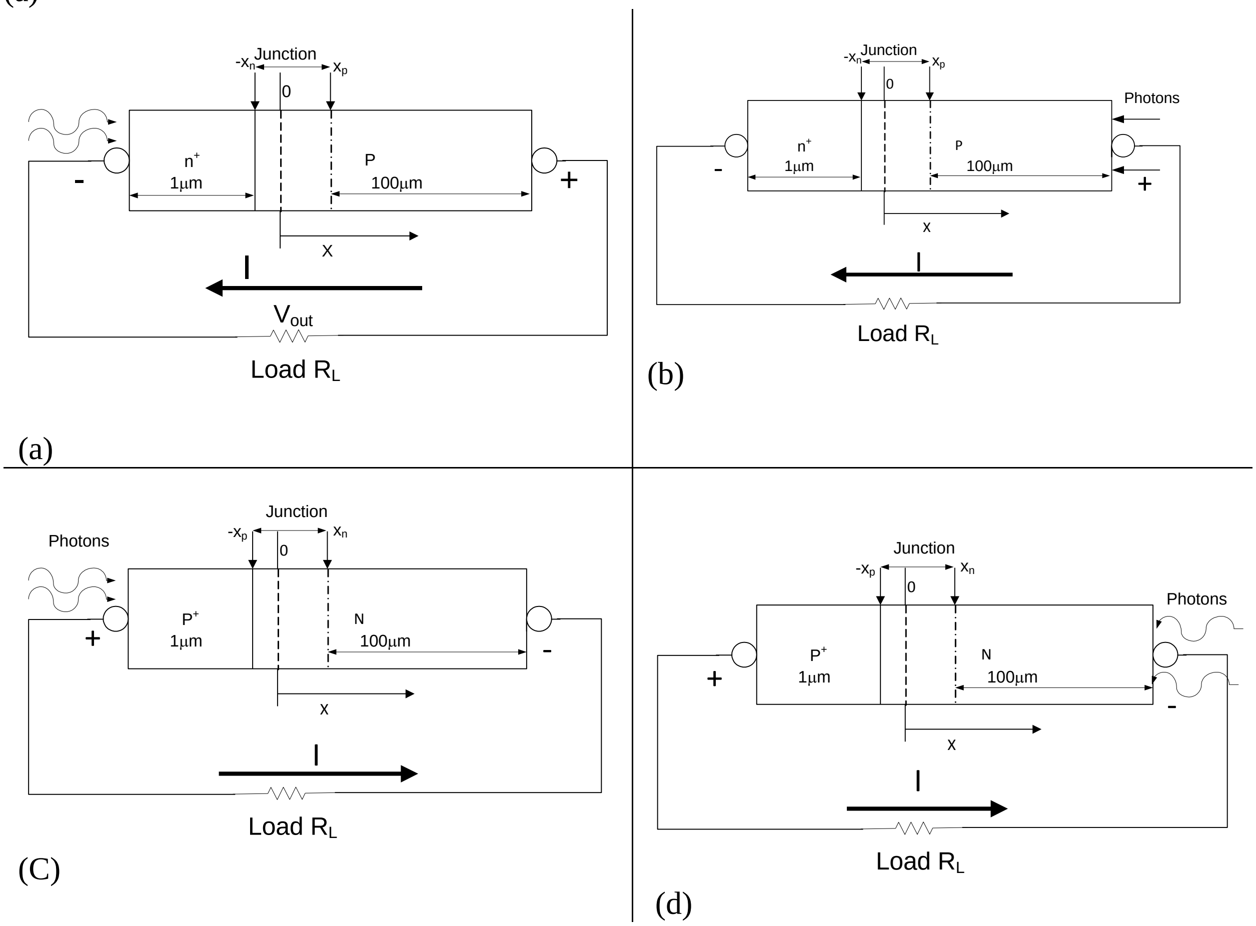
Problem (c)

Fill Factor (FF) = $\dfrac{V_m I_m}{V_{OC} I_{SC}} = \dfrac{0.59 * 25.87mA}{0.6721 * 27mA} = 0.8437$

Problem (d)
Voc Increase
FF   Increase

Problem (e)
Decrease

Q.3(a) Show the polarity of output voltage and direction of current in load R_L of all solar cells in Fig. 3.
Solution:
(a)



Q3(b) η(a) > η(c) > η(b) > η(d) (Efficiency of the cells in the order).
Q3(c) Cell (a) has maximum output.

Q.4. What does I_{sc} and V_{oc} primarily depend on: Circle the right answers
I_{sc} depends on (a) band gap, (b) very slightly on doping levels, (c) thickness of the n-region in a p-n cell, and (d) solar power incident on the cell.

V_{oc} depends on all four factors listed. It also depends on temperature.

Q.5. Figure 4 shows four losses in a solar cell on the bar chart.
(a) Show that the long wavelength loss is 176.86 W/m^2 [(925 -748.14) as shown below in Fig. 5] for Air Mass m = 1 condition.

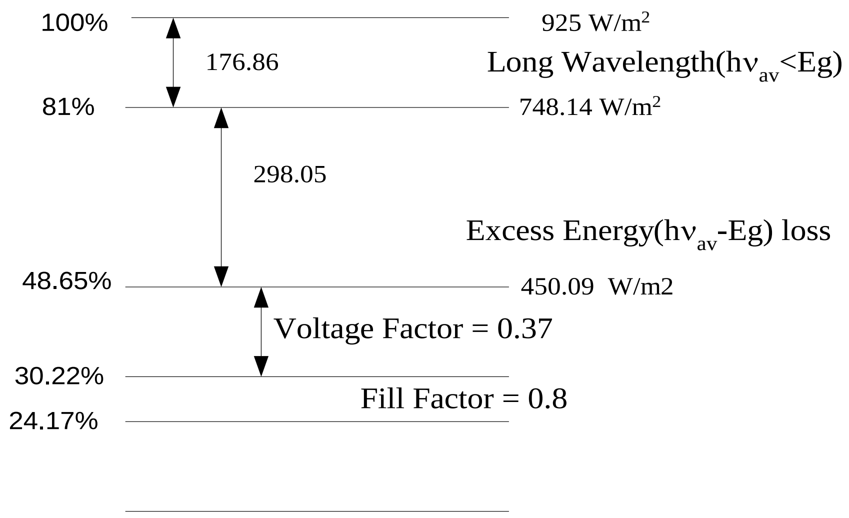


Fig. 5. Four losses in a solar cell receiving 925W/m^2 at an Air Mass m =1.

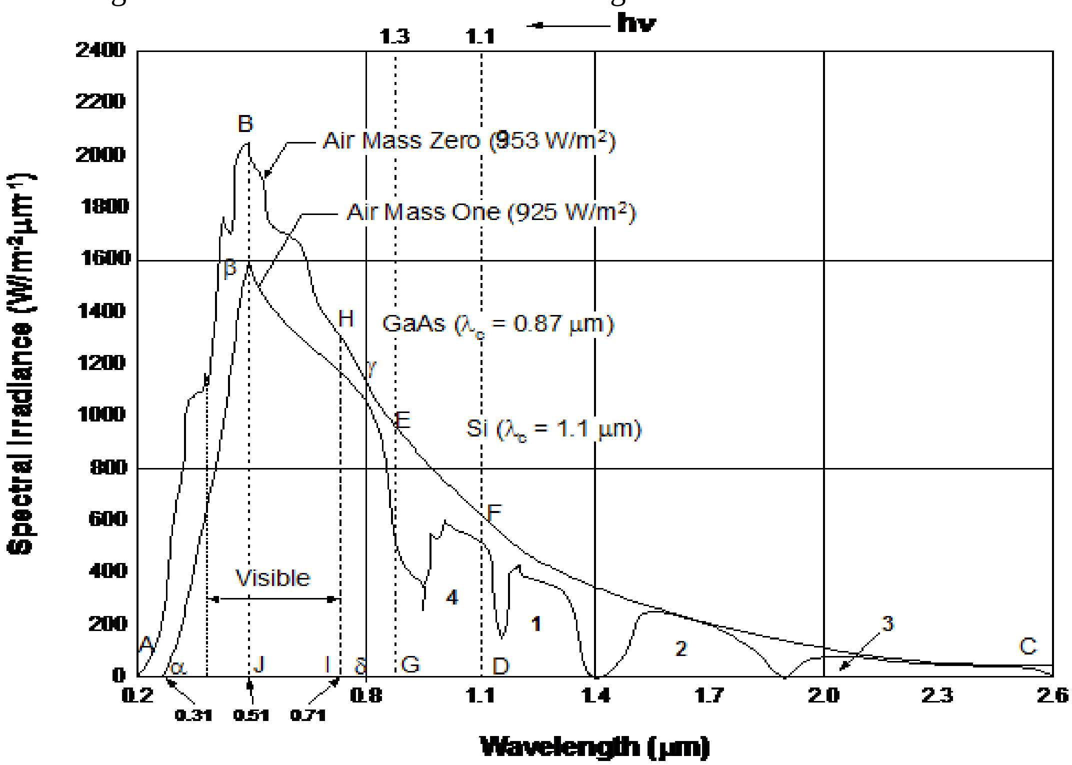


Fig. 4. Solar spectrum under AM O and AM1.

Solution:
(a) Long wavelength losses for AM1

With reference to the above figure, in area of region 1, 2 & 3, the solar power is:

Region #1 = 0.5*456*0.3 = 68.4 W/m^2
Region #2 = 0.5*297.6*0.5 = 74.4W/m^2
Region #3 = 51.5*0.7 = 36.05 W/m^2

Total long wavelength photons loss = 68.4+72.69+35.77 = 176.86 W/m^2

(b) Calculate the excess energy lost in the spectral range represented by triangle αβJ for AM1.
The area of triangle αβJ = [1600*(0.51 – 0.31)]/2 = 160 W/m^2.
The average photon energy $h\nu_{av}$ in triangle αβJ is

(1.24/0.31 + 1.24/0.51)/2 = (4.0 + 2.43)/2= 6.43/2 = 3.215 eV.

Excess energy per photon not used in creating electron-hole pairs in Si (E_g=1.1eV) is

3.215-1.1 = 2.115eV.

Excess energy not converted into electron-hole pairs and is lost to heat in triangle αβJ is

(160/3.215)*2.115=105.25 W/m^2

Correction: The value listed in this question of 298.5 W/m2 is the total excess energy loss in the entire spectrum, not in triangle αβJ.

Q.6(a). How do the excess carriers recombine in forward biased Si diode Fig.5(a) and GaAs diode of Fig. 5b (whose energy band diagram is shown in Fig. 6, respectively.)?

In pSi: The injected electrons in p-Si side $n_p(x)$ recombine creating photons. However, electron momentum (related to k_c) in conduction band valley is different from holes at k_v in the valance band. To conserve the momentum we need the assistance of phonons. They have small energy ~20 milli electron Volt (or 0.020eV) and momentum that is comparable with that of electrons and holes. The equations are:

$$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\,hole} + (h/2\pi)\, k_{photon} + (h/2\pi)\, k_{phonon} = 0 \qquad \text{(F)}$$

Neglecting photon momentum which is very small, we get

$$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\,hole} + (h/2\pi)\, k_{phonon} = 0 \qquad \text{(G)}$$

The sign of phonon momentum depends if a phonon is generated or absorbed.

Figure 2 (left) below shows when a phonon is created. In this case the photon energy

$$h\nu = E_g - E_{phonon} = 1.1\text{eV} - 0.020 = 1.08\text{eV}.$$

The transition when a phonon is absorbed is

$$h\nu = E_g + E_{phonon} = 1.1\text{eV} + 0.020 = 1.12\text{eV}.$$

In pGaAs: The electrons are injected from NAlGaAs into p-GaAs. GaAs is a direct gap material and hence electrons in the conduction band recombine with holes in the valance band. They do not require the assistance of phonons to conserve momentum.

$$h\nu = E_g = 1.424\text{eV}$$

The momentum conservation is $(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\,hole} = 0$

(b) Mark the region or regions where photons are created.

(c) What is the distance from junction edge (x_p) in which electron concentration decays by 1/e?

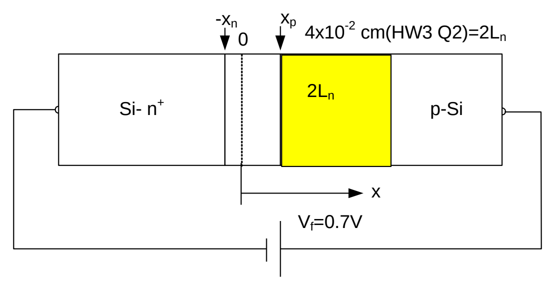


Fig. 5(a) A forward biased n+-p Si diode.

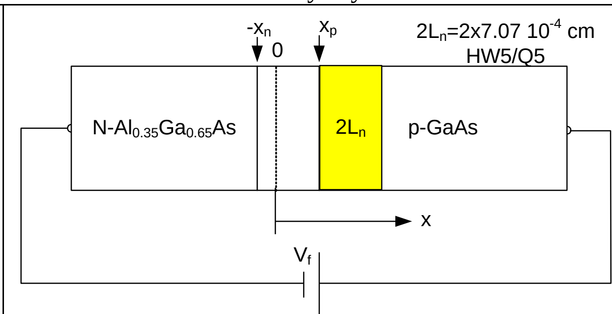


Fig. 5(b). A forward biased nAlGaAs-p GaAs diode.

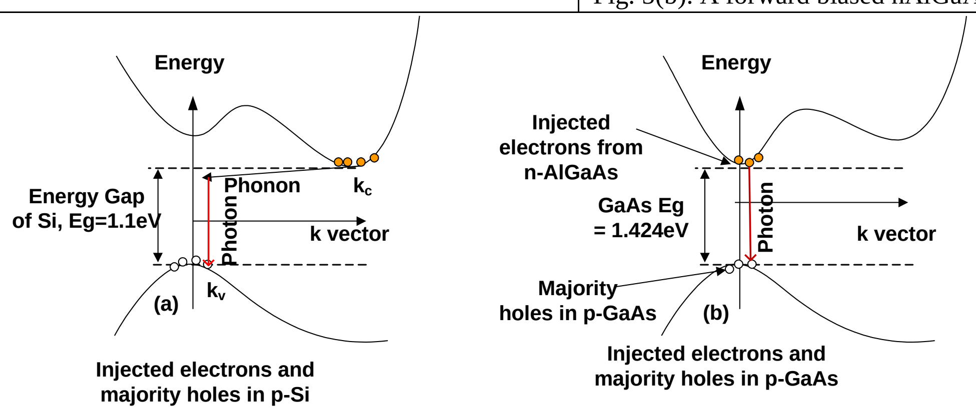


Fig. 6 shows the energy of injected electrons at a point x on the p-side.

Q.7. (a) As shown above in Q.6(a) In Si there is participation of phonons (E_{phonon} ~0.02eV). In this case the photon energy in the presence of phonon creation or emission

$$h\nu = E_g - E_{phonon} = 1.1\text{eV} - 0.020 = 1.08\text{eV}.$$

The transition when a phonon is absorbed is

$$h\nu = E_g + E_{phonon} = 1.1\text{eV} + 0.020 = 1.12\text{eV}.$$

In Si, the conservation of momentum can be seen from Fig. 6 (left)

$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\, hole} + (h/2\pi)\, k_{photon} + (h/2\pi)\, k_{phonon} = 0$ (F)

Neglecting photon momentum which is very small, we get

$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\, hole} + (h/2\pi)\, k_{phonon} = 0$

In GaAs the photon energy is E2-E1~ E_g = 1.424eV. Here, E2 is upper level where an electron is and E1 is the lower level where the hole is.

The momentum conservation is:

$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\, hole} + (h/2\pi)\, k_{photon} = 0$

Since photon momentum is negligible,

$(h/2\pi)\, k_{c,elec} + (h/2\pi)\, k_{v,\, hole} = 0$

(b) If the quantum efficiency η_q for GaAs is 0.95 and for Si 0.05 how many photons are produced per second for an electron current of 1mA in these devices on the p-side.

Photons produced per second by 1mA in GaAs = $[10^{-3}/q]^* \eta_q$=

$=(10^{-3} *0.95)/1.6*10^{-19} = 5.9*10^{15}$ photons/s

Photons produced per second by 1mA in Si = $[10^{-3}/q]^* \eta_q$= = $(10^{-3} *0.05)/1.6*10^{-19} = 3.33*10^{14}$.

Q.8 (a) and (b). Find the composition of InGaAs active layer to design LEDs operating at 1.35 and 1.55 microns using band gap-lattice parameter data shown in Fig. 7.

The energy of photon at wavelength of 1.3 micron is 0.919eV (~0.92eV) and 1.55 micron is 0.8eV (using 1.24/1.55 micron).

Draw a horizontal line from ~0.92eV and 0.8eV. Since the substrate corresponds to InP, we draw a vertical line from InP. The 0.8eV line intersects at C, and 0.92 intersects at Point B, and the vertical line intersection with GaAs-InAs line at A. Composition of point A is $In_{0.528}Ga_{0.472}As$. The composition of points C and B will also have phosphorus as their composition is in between $In_{0.528}Ga_{0.472}As$ and InP. [REF: See ECE 4211 Notes] At C:

Indium fraction at C= Indium at A + fraction (AC/A-InP)*0.472=0.528+(5/12)*0.472 = 0.724; Ga is = 1- Indium= 1- 0.724= 0.276. Note that we measured fraction (AC/A-InP) = 5/12=0.416. Phosphorus composition at point C = Phosphorus at A (zero) + fraction (AC/A-InP)*1= 0+ 0.416=0.416; the Arsenic composition is = 1- Phosphorus composition= 1-0.416 = 0.584.

Point C = $In_{0.724}Ga_{0.276}As_{0.584}P_{0.416}$

Similarly, find fraction of In and P for point B to compute the composition of InGaAsP.

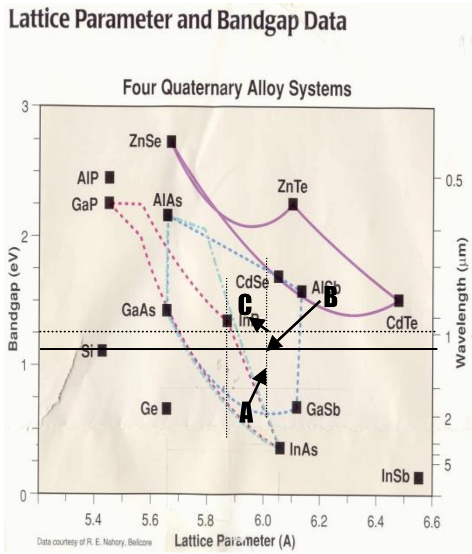


Fig. 3a (left)

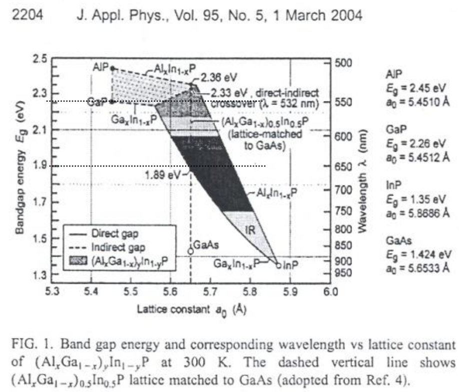


Fig. 3b Lattice constant versus energy gap.

Q.8(c) Name materials which will emit light at 1.55 microns, 6500A, 5500A, 5850A, 5050A, and 4600A. Use the energy gap vs lattice constant diagrams given above.

Answer: 1.55 micron is done above. For 5850A (=0.585 microns), 5500A and 6500A, draw horizontal lines from Y-or Energy Axis at 2.11eV, 2.25eV and 1.9eV. Since the substrate corresponds to GaAs in Fig. 3b, we draw a vertical line from GaAs (this line is already there).

Q. 9. An n^+-p $GaAs_{0.6}P_{0.4}$ homojunction diode is schematically shown in Fig.8.

(a) **Calculate** I_n $(x=x_p)$ and I_p $(x=-x_n)$ at a forward bias of V_f =1.55 Volt.

(b) **Find** number of photons generated per second in the p-region.

(c) **Determine** the wavelength of light generated in the medium and as viewed in air.

(d) **Identify** the location and length of the region where most of the photons are generated.

(e) Calculate the internal efficiency η_{int}. at forward bias V_f of 1.55 volts and 1.5V.

HINT: $\eta_{int} = \eta_q * \eta_{inj}$ *Given--quantum efficiency in* $GaAs_{0.6}P_{0.4}$ is $\eta_q = 0.25$.

$$\eta_{inj} = \frac{I_n(x_p)}{I_n(x_p)+I_p(-x_n)} = \frac{\left(\frac{qAD_n n_{po}}{L_n}\right)\left(e^{\frac{qV_f}{kT}}-1\right)}{\left(\frac{qAD_n n_{po}}{L_n}+\frac{qAD_p p_{no}}{L_p}\right)\left(e^{\frac{qV_f}{kT}}-1\right)} = \frac{\frac{(D_n n_{po})}{L_n}}{\frac{(D_n n_{po})}{L_n}+\frac{(D_p p_{no})}{L_p}}$$

(f) Find the extraction efficiency $\eta_{extraction}$.

$$\eta_{extraction} = \frac{1}{2}\bullet\frac{4n_r}{(1+n_r)^2}\bullet\left[1-\left(1-\frac{1}{n_r^2}\right)^{\frac{1}{2}}\right]$$

The device parameters are: **n+ side**	**p-side**
N_D = donor concentration=$1x10^{18}$ cm^{-3}	Acceptor concentration N_A=$1x10^{16}$ cm^{-3}
Minority hole lifetime τ_p =$5x10^{-9}$ sec	Minority electron lifetime τ_n =$1x10^{-8}$ sec
Hole diffusion coefficient D_p=10 cm^2/s	Electron diffusion coefficient D_n=50 cm^2/s

Assume all donors and acceptors to be ionized at 300K.

$GaAs_{0.6}P_{0.4}$ energy gap E_g= 1.85 eV
Band gap type = direct, Intrinsic concentration at 300K n_i = 300 cm^{-3}
n_i at 500K = $1x10^9$ cm^{-3},
Junction area A = $1x10^{-3}$ cm^2,
Dielectric constant ε_r =12.84, Free space permittivity ε_o =$8.854x10^{-14}$ Farad/cm,
Temperature T =300 K unless specified.
Index of refraction, n_r, of $GaAs_{0.6}P_{0.4}$ =3.58 Quantum efficiency η_{q}= 0.25.
Use index of refraction equation.
Electron effective mass m_n = 0.067 m_o, hole effective mass = 0.62m_o

Electron charge q=$1.6x10^{-19}$ Coulombs, Boltzmann constant k=$1.38x10^{-23}$ J/K

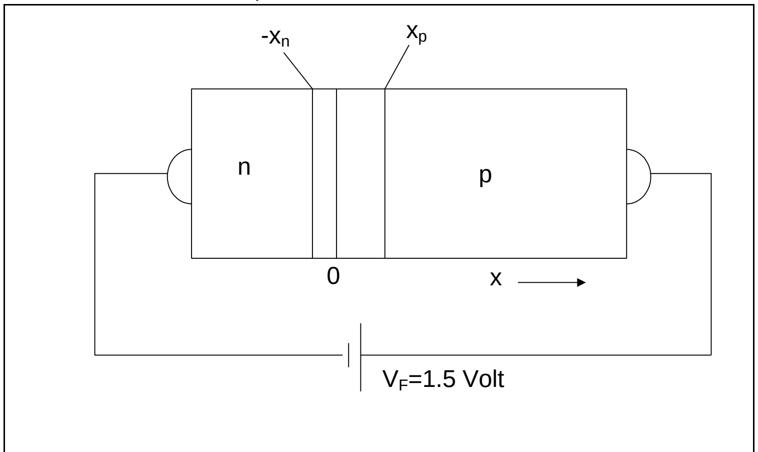


Fig. 8. A forward biased $nGaAs_{0.6}P_{0.4}$ - $pGaAs_{0.6}P_{0.4}$ diode.

Solution Q9. (a)

$$I_n(x = x_p) = \frac{qAD_n n_{po}}{L_n}\left(e^{\frac{qV_f}{kT}} - 1\right)$$

$$= \frac{1.6\times10^{-19}\times10^{-3}\times50\times9\times10^{-12}}{7.07\times10^{-4}}\left(e^{\frac{1.55}{0.0259}} - 1\right)$$

$$= 1.6\times10^{-22}\times6.36\times10^{-7}\times(9.786\times10^{25} - 1)$$

$$\Rightarrow I_n(x = x_p) = 9.958\times10^{-3}\,Amp$$

$$n_{po} = \frac{n_i^2}{p_{po}} = \frac{n_i^2}{N_A} = \frac{9\times10^4}{10^{16}} = 9\times10^{-12}cm^{-3}$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_p} = \frac{9\times10^4}{10^8} = 9\times10^{-14}cm^{-3}$$

$$L_n = \sqrt{D_n\tau_n} = \sqrt{50\times10^{-8}} = 7.07\times10^{-4}\,cm$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10\times5\times10^{-9}} = 2.23\times10^{-4}\,cm$$

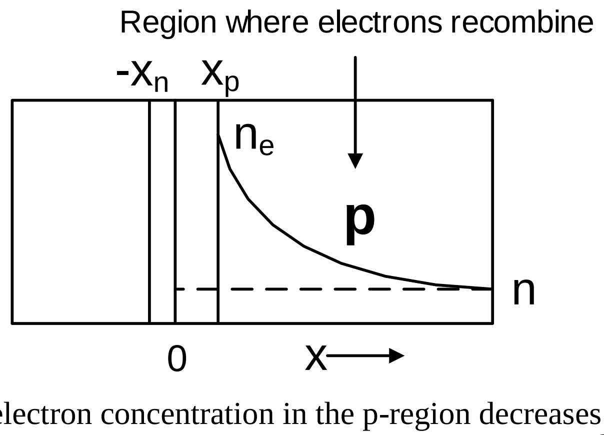
$$I_p(x = -x_n) = \frac{qAD_p p_{no}}{L_p}\left(e^{\frac{qV_f}{kT}} - 1\right)$$

$$= \frac{1.6\times10^{-19}\times10^{-3}\times10\times9\times10^{-14}}{2.23\times10^{-4}}\left(e^{\frac{1.55}{0.0259}} - 1\right)$$

$$= 1.6\times10^{-22}\times4.035\times10^{-9}\times(9.786\times10^{25} - 1)$$

$$I_n(x = -x_n) = 6.3\times10^{-5}\,Amp$$

(b)



The electron concentration in the p-region decreases by a factor of 1/e, a distance of L_n=7.07 x 10^{-4} cm from x_p. The concentration decreases by $1/e^2$ in a distance of $2L_n$=1.41 x 10^{-3} cm (b)Number of photons generated per second in the p-region

$$= \frac{I_n(x_p)}{q} \times \eta_q = \frac{9.958\times10^{-3}}{1.6\times10^{-17}} \times 0.25$$

Number of photons generated /sec

$$= 1.55\times10^{16} / \mathrm{sec}$$

(c) Energy of photons

$$E_g \cong h\nu = 1.85eV$$

$$\lambda = \frac{c}{\nu}$$

$$\lambda = \frac{c}{E_g/h} = \frac{hc}{E_g} = \frac{1.24}{1.85eV}(\mu m)$$

$$= 0.6707\mu m = 6702\mathrm{A}$$

In the medium (or semiconductor layer)

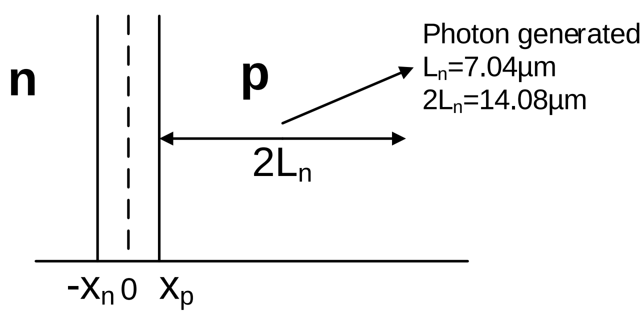
$$\lambda_{medium} = \frac{\lambda_{air}}{n_r} = \frac{\lambda}{n_r} = \frac{0.6702}{3.58} = 0.1872\mu m$$

$$n_r = \sqrt{\varepsilon_r} = \sqrt{12.84} = 3.58$$

(d) Number of photons generated in the p-region: Where?

The number goes by 1/e in a distance equal to the diffusion length L_n and $1/e^2$ in a distance from junction boundary.

2 x L_n = 2 x 7.07 µm



(e) Internal efficiency is $\eta_{\mathrm{int}} = \eta_{inj} * \eta_q$

For 1.55 Volt forward bias

$$\eta_{inj} = \frac{I_n(x_p)}{I_n(x_p) + I_p(x_n)} = \frac{9.958\times10^{-3}}{9.958\times10^{-3} + 6.3\times10^{-5}} = \frac{9.958\times10^{-3}}{10\times10^{-3}}$$

$$= 0.99371$$

$$\eta_{\mathrm{int}}\Big|_{V_F = 1.55Volt} = 0.99371 * 0.25 = 0.248425 \Leftarrow$$

For 1.5 Volt

$$I_n(x_p) = \frac{qAD_n n_{po}}{L_n}\left(e^{\frac{qV_f}{kT}} - 1\right)$$

$$= \frac{1.6\times10^{-19}\times10^{-3}\times50\times9\times10^{-12}}{7.07\times10^{-4}}\left(e^{\frac{+1.5}{0.0259}}-1\right)$$

$$= \frac{1.6\times10^{-19}\times10^{-3}\times50\times9\times10^{-12}}{7.07\times10^{-4}}\left(1.42\times10^{25}-1\right)$$

$$I_n(x_p)=1.44\times10^{-3}\,Amp$$

$$Similarly,$$

$$I_p(-x_n)=\frac{1.6\times10^{-19}\times10^{-3}\times10\times9\times10^{-14}}{2.23\times10^{-4}}\left(1.42\times10^{25}-1\right)$$

$$I_p(-x_n)=9.16\times10^{-6}\,Amp$$

$$\eta_{inj}=\frac{1.44\times10^{-3}}{1.44\times10^{-3}+9.16\times10^{-6}}=0.99368$$

$$\eta_{\text{int}}\Big|_{V_F=1.5Volt}=0.99368*0.25=0.24842\Leftarrow$$

Pretty much the same value of internal efficiency η_{int} at two biases.

Optional: Extraction efficiency

$$\eta_{extraction}=\frac{1}{2}\bullet\frac{4n_r}{(1+n_r)^2}\bullet\left[1-\left(1-\frac{1}{n_r^2}\right)^{\frac{1}{2}}\right]\qquad=\frac{1}{2}\bullet\frac{4*3.58}{(4.58)^2}\bullet\left[1-\left(1-\frac{1}{3.58^2}\right)^{\frac{1}{2}}\right]$$

$$=\frac{1}{2}\bullet0.68\bullet[1-0.96]\qquad=\frac{1}{2}\bullet0.68\bullet[0.04],\ \eta_{extraction}=0.0136\quad or\quad 1.36\%.$$

Summary of losses

1. Surface reflection from the semiconductor-air boundary.
2. Total internal reflection of photons with incident angle θ_i > critical angle θ_c.
3. 50% photons are lost as they travel towards the back contact.

Q. 10 Layers to reduce losses (see Fig. 9)

1. Have an antireflection coating between the dome and LED p-n device
   ➔ Find n_{r2} of antireflection coating and find the thickness t.
2. Increase critical angle value by replacing air medium by epoxy dome (layer n_{r3}) which has a higher index of refraction n_{r3} than air which is 1. The new critical angle is $\sin^{-1}(n_{r3}/n_r)$.
3. Have a lower index of refraction layer (n_{r1}) below the semiconductor region (with index n_r) where photons are generated. This layer will reflect the back side traveling photons by introducing total internal reflection. (like on the front side).

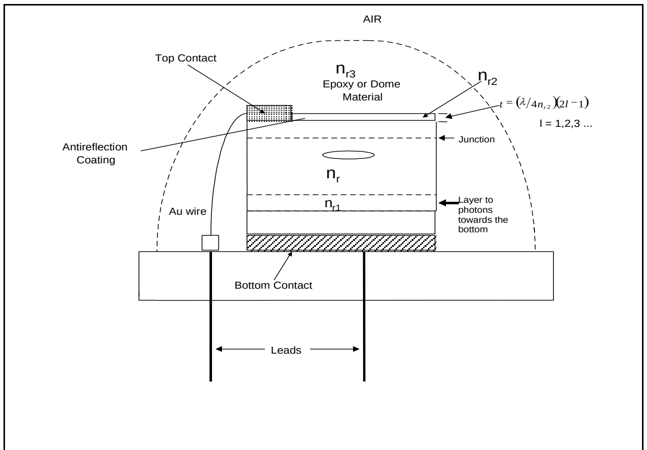


Fig. 9 An encapsulated LED showing layers reducing losses and improving extraction efficiency.

Q.1(a) What is an exciton and how does it form in GaP doped with nitrogen? Is nitrogen atoms behave as donors or acceptors in GaP?
HINT: Exciton is an electron-hole pair with hole forming the core and electron revolving around it, like electron orbiting in a hydrogen atom.

Answer: Nitrogen is from group V and is has the same number of valence electrons as P. So it does not behave as a donor or acceptor. It introduces an energy level at 0.08eV below the conduction band. Injected electrons fall to this level and at the same time form a meta-stable pair with hole belonging to the valence band. Since excitons are very well confined at the location of Nitrogen atoms, their position is relatively fixed. As a result due to following condition, their momentum is fluctuations are large. This assist in conserving momentum condition, and the exciton transition takes place more efficiently in the indirect gap GaP p-region.

$$\Delta x \, \Delta p > h/2 \text{ (Heisenberg uncertainty principle)}$$

Why an n-p GaP diode with p-region having Zn and nitrogen doping gives efficient green light than the same diode without nitrogen doping? Shown below is another example of exciton formation. Here, where there is Zn-O site, an exciton forms. Similarly, one can envision, nitrogen atoms replacing P.

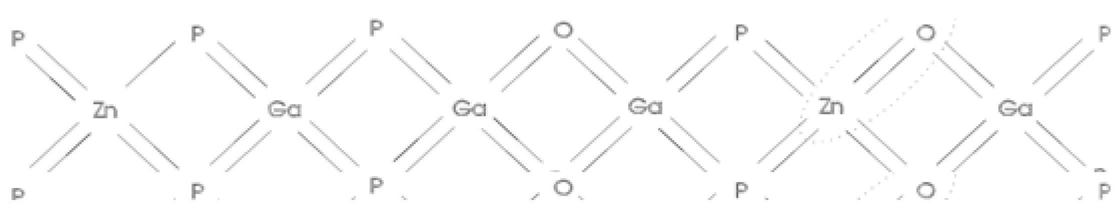


Fig. 10 Doping of GaP with Zn-O.

Formation of excitons:

What are the energies of photons with N and without N doping in GaP LED.

In this case the p-side of GaP is doped both with Zn as well as nitrogen shown in Fig. 10. Nitrogen is pentavalent like phosphorus, therefore, the substitution of a phosphorus atom by a nitrogen atom does not create a donor or an acceptor. However, the addition of N introduces a neutral energy level 0.08 eV below the conduction band edge. This is shown in Figure 11.

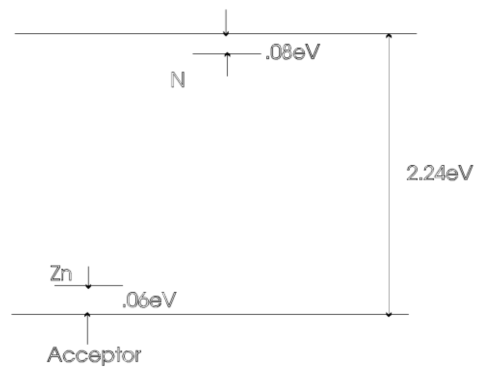


Figure 10. Energy levels in a GaP: N, Zn system

Exciton formation

An injected electron gets trapped at the nitrogen level, and subsequently binds a hole to form an exciton. The decay of this exciton results in a photon emission. The energy of the photon is

$$h\nu = E_g - 0.08 - E_{ex} \qquad (6)$$

The electron-hole recombination via excitonic decay is almost like a vertical (direct) transition.
If E_{ex} = 0.004eV

$$h\nu = 2.24 - .08 - .004 = 2.156eV \qquad (7)$$

$$\lambda = \frac{1.24}{2.156} = 0.575\mu m = 5750\text{A}$$

Solutions to other parts will be provided after the submission.