

VLSI Design and Simulation

Lecture 3

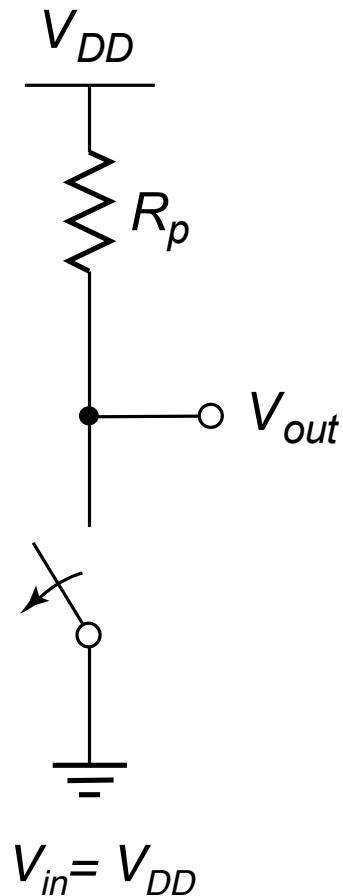
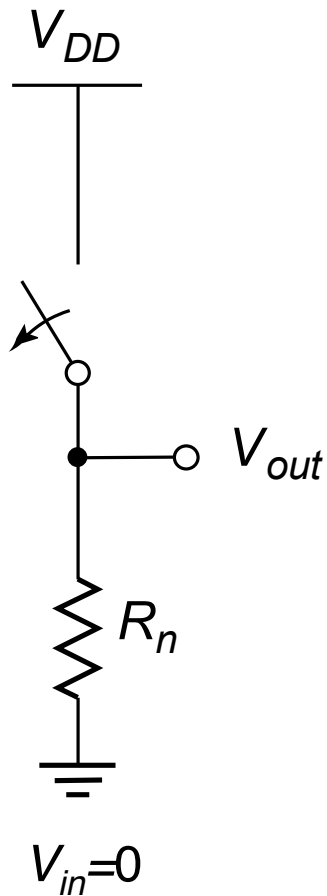
CMOS Inverters

Topics

- Inverter VTC
- Noise Margin
- Static Load Inverters

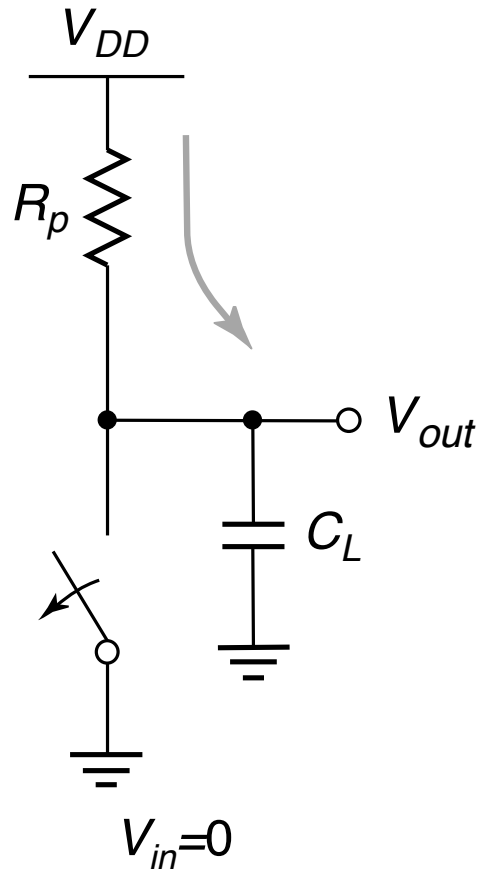
CMOS Inverter

First-Order DC Analysis

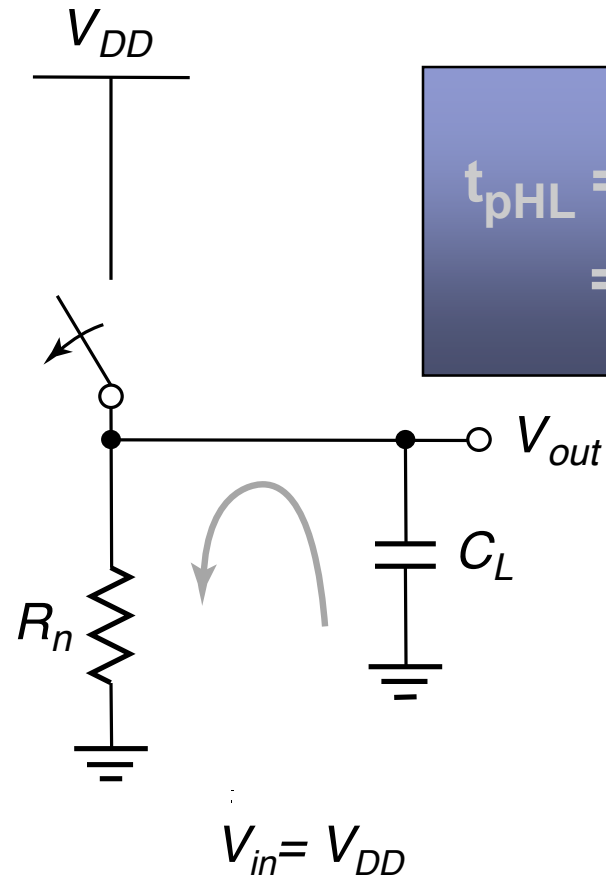


$$V_{OL} = 0$$
$$V_{OH} = V_{DD}$$

CMOS Inverter: Transient Response



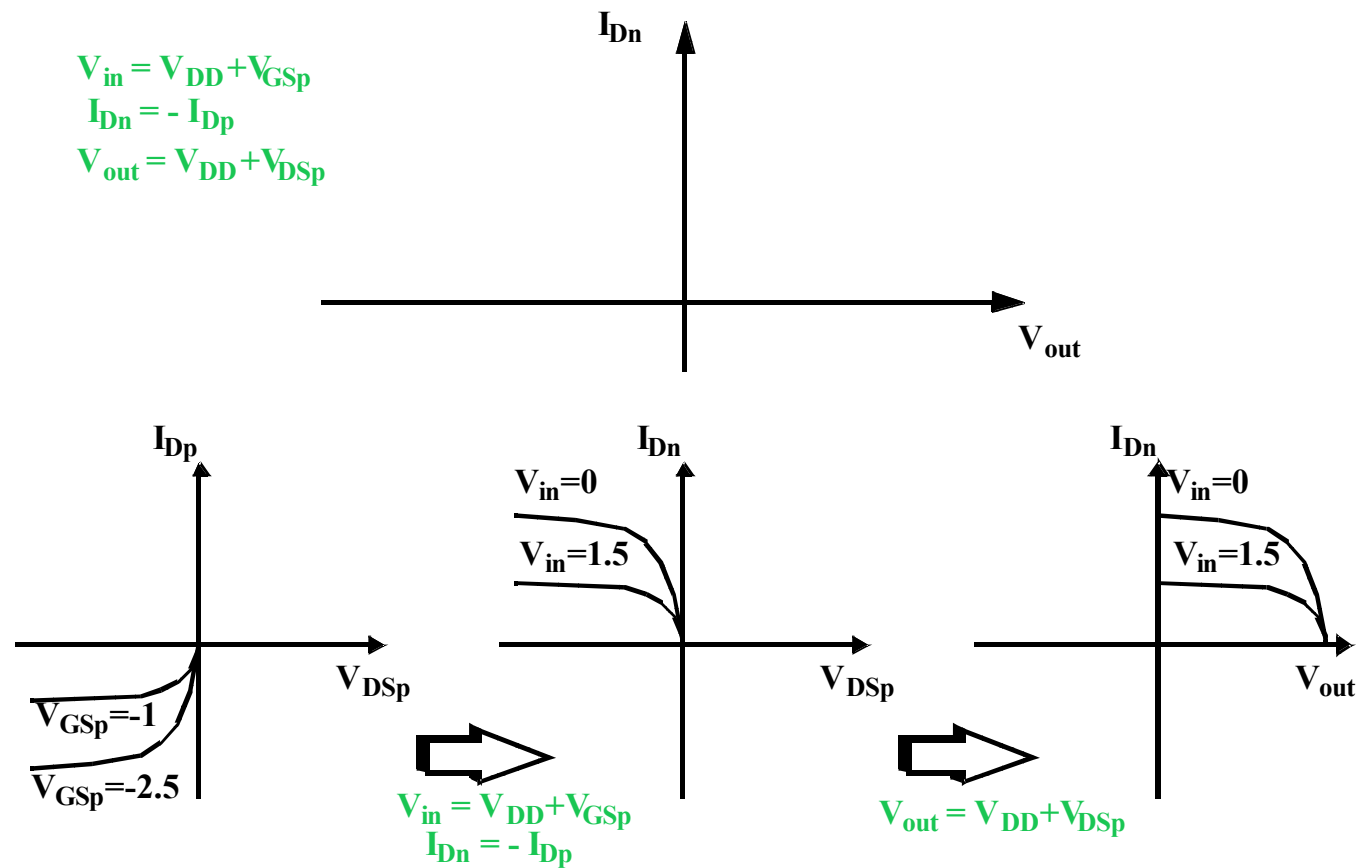
(a) Low-to-high



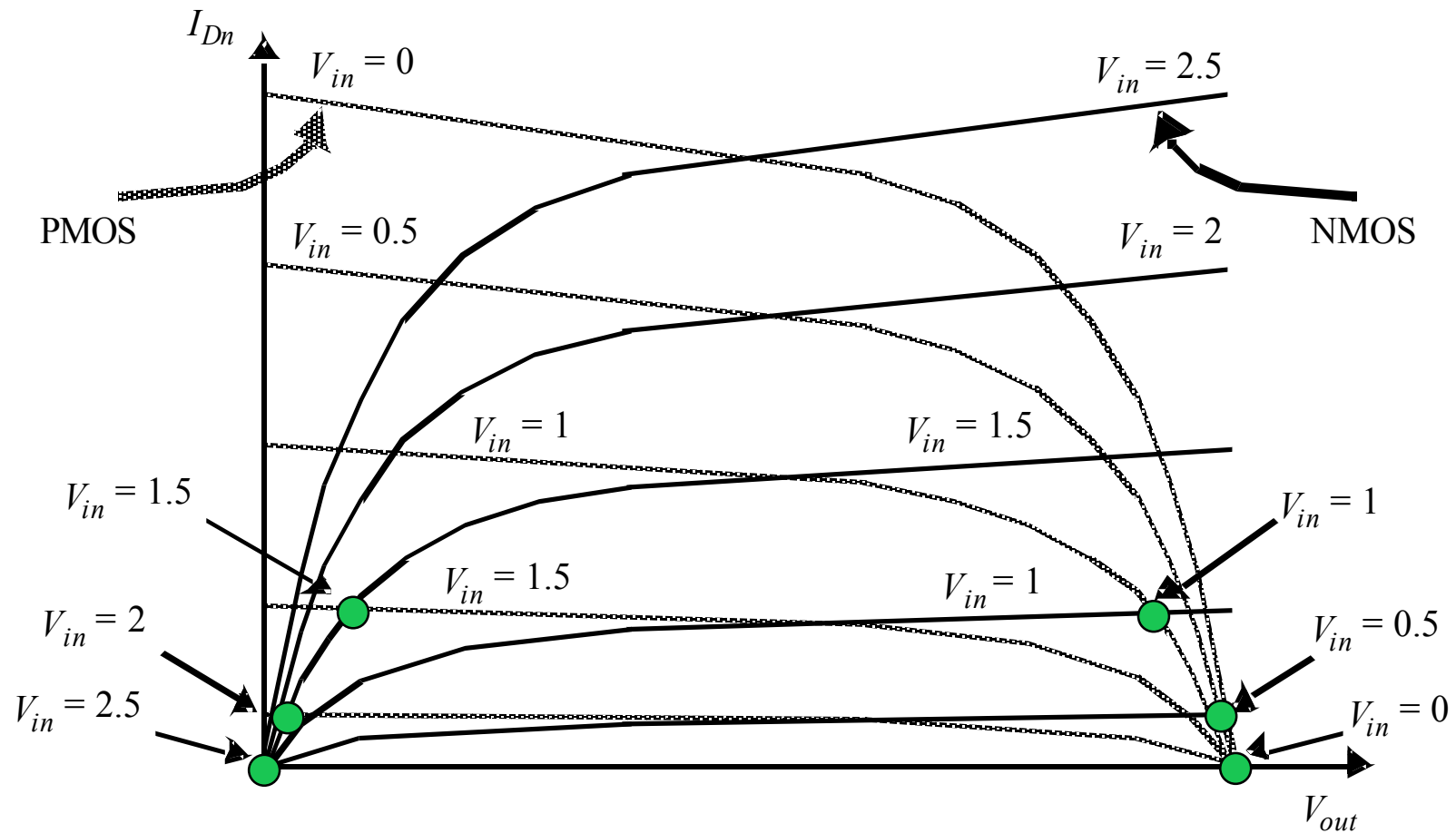
(b) High-to-low

$$t_{pHL} = f(R_{on} \cdot C_L) \\ = 0.69 R_{on} C_L$$

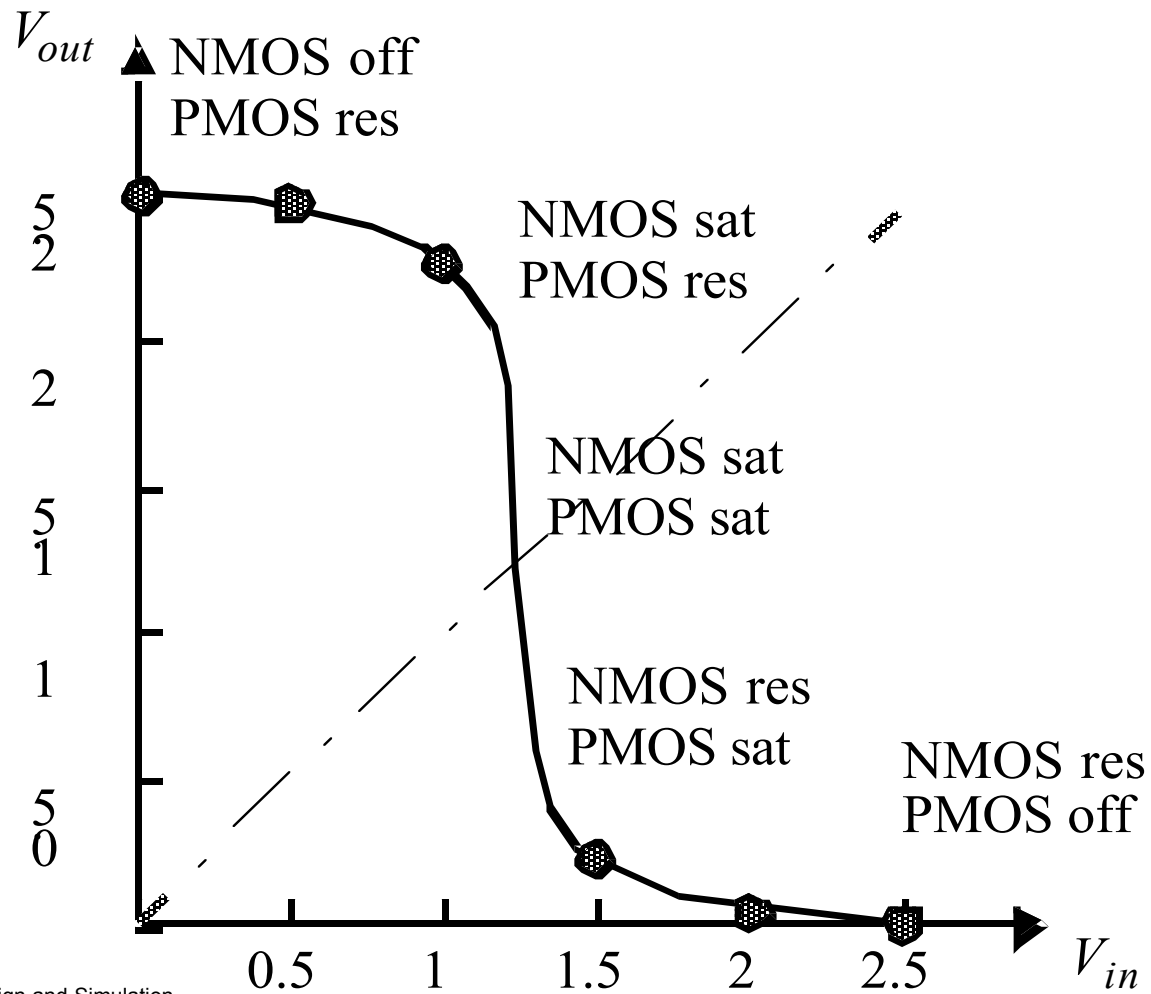
PMOS Load Lines



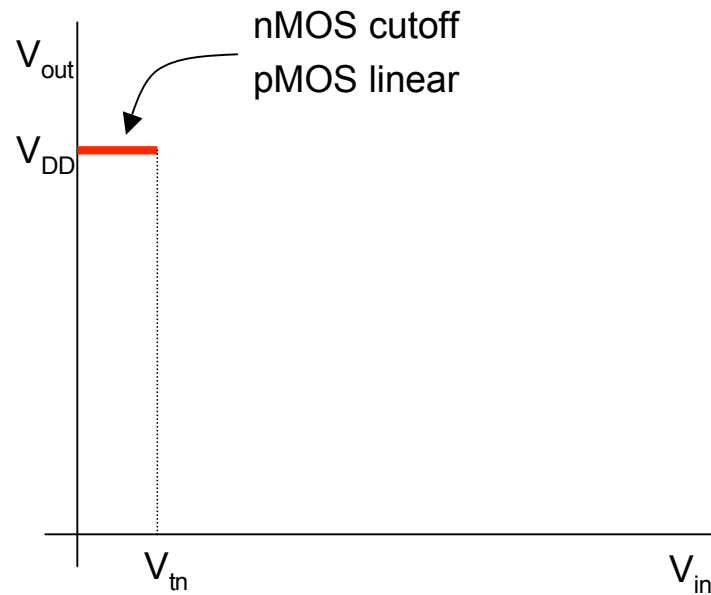
CMOS Inverter Load Characteristics



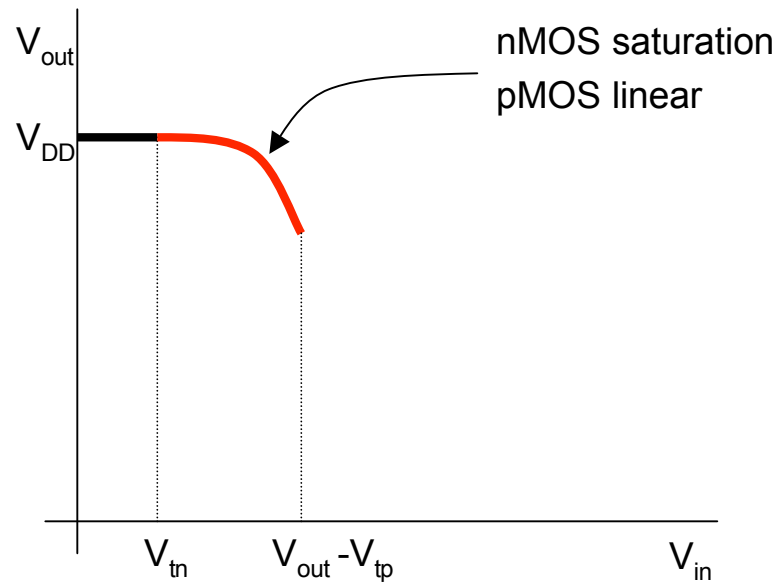
CMOS Inverter VTC



CMOS Inverter VTC



CMOS Inverter VTC



CMOS Inverter VTC

- Set pMOS Linear I_{DS} equal to nMOS Saturation I_{DS}

$$k_n \left(\frac{(V_{in} - V_{tn})^2}{2} \right) = k_p \left((V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) - \frac{(V_{out} - V_{DD})^2}{2} \right)$$

$$\frac{(V_{out} - V_{DD})^2}{2} - (V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) + \frac{k_n}{k_p} \frac{(V_{in} - V_{tn})^2}{2} = 0$$

$$(V_{out} - V_{DD}) = (V_{in} - V_{DD} - V_{tp}) + \sqrt{(V_{in} - V_{DD} - V_{tp})^2 - \frac{k_n}{k_p} (V_{in} - V_{tn})^2}$$

$$V_{out} = (V_{in} - V_{tp}) + \sqrt{(V_{in} - V_{DD} - V_{tp})^2 - \frac{k_n}{k_p} (V_{in} - V_{tn})^2}$$

CMOS Inverter VTC

- Short channel model

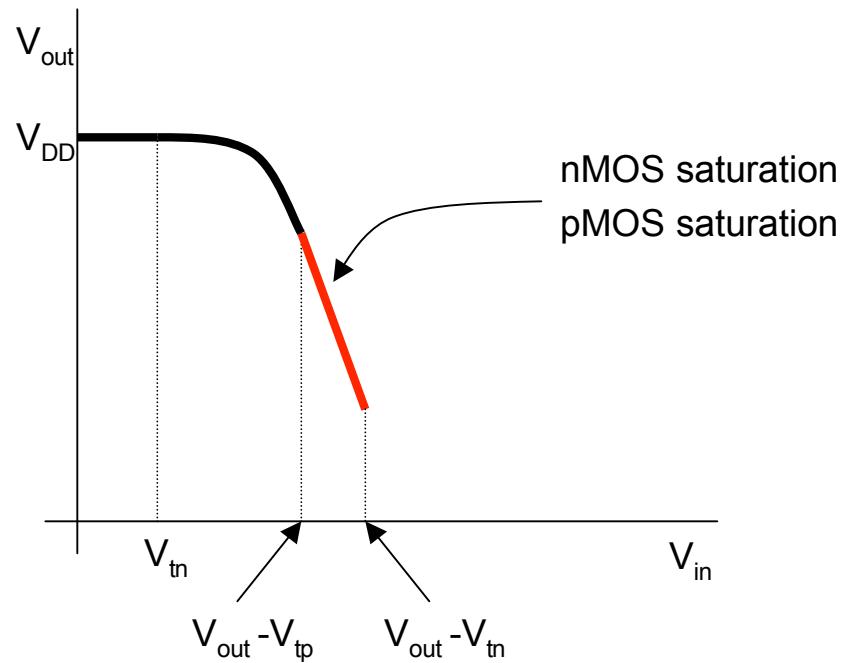
$$k_n V_{DSATn} \left(V_{in} - V_{tn} - \frac{V_{DSATn}}{2} \right) = k_p \left((V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) - \frac{(V_{out} - V_{DD})^2}{2} \right)$$

$$\frac{(V_{out} - V_{DD})^2}{2} - (V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD}) + \frac{k_n}{k_p} V_{DSATn} \left(V_{in} - V_{tn} - \frac{V_{DSATn}}{2} \right) = 0$$

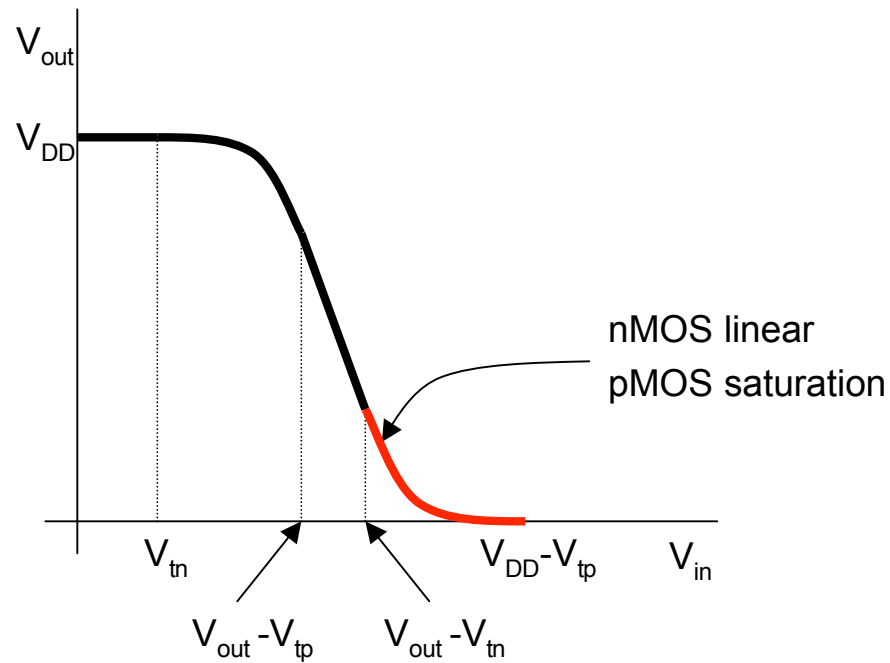
$$(V_{out} - V_{DD}) = (V_{in} - V_{DD} - V_{tp}) + \sqrt{(V_{in} - V_{DD} - V_{tp})^2 - 2 \frac{k_n}{k_p} V_{DSATn} \left(V_{in} - V_{tn} - \frac{V_{DSATn}}{2} \right)}$$

$$V_{out} = (V_{in} - V_{tp}) + \sqrt{(V_{in} - V_{DD} - V_{tp})^2 - 2 \frac{k_n}{k_p} V_{DSATn} \left(V_{in} - V_{tn} - \frac{V_{DSATn}}{2} \right)}$$

CMOS Inverter VTC



CMOS Inverter VTC



CMOS Inverter VTC

- Set nMOS Linear I_{DS} equal to pMOS Saturation I_{DS}

$$k_p \left(\frac{(V_{in} - V_{DD} - V_{tp})^2}{2} \right) = k_n \left((V_{in} - V_{tn})V_{out} - \frac{V_{out}^2}{2} \right)$$

$$\frac{V_{out}^2}{2} - (V_{in} - V_{tn})V_{out} + \frac{k_p}{k_n} \frac{(V_{in} - V_{DD} - V_{tp})^2}{2} = 0$$

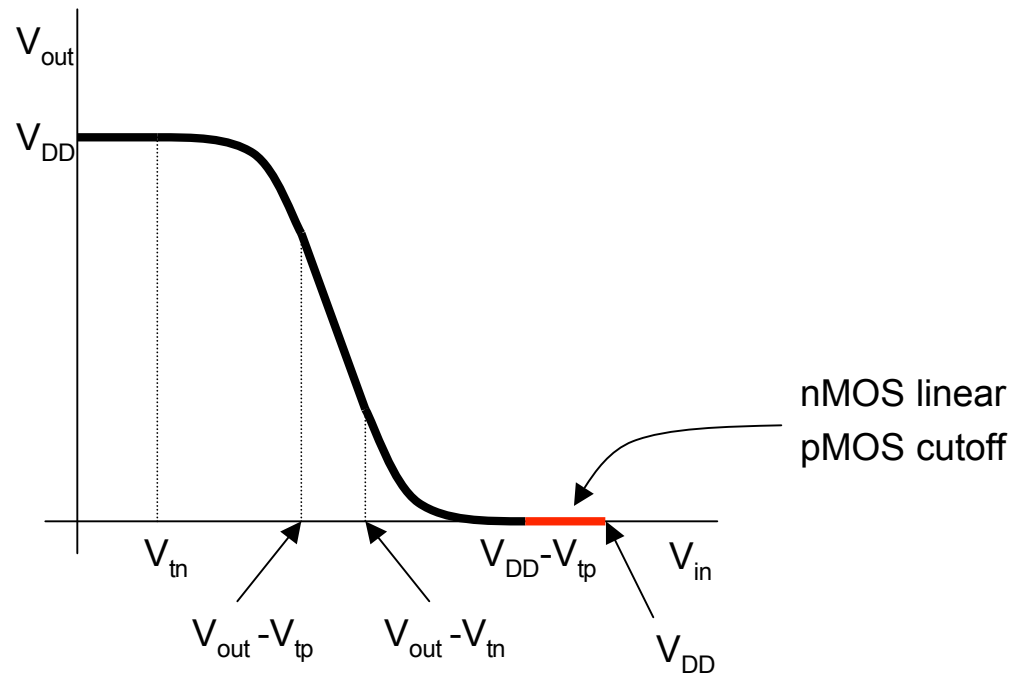
$$V_{out} = (V_{in} - V_{tn}) - \sqrt{(V_{in} - V_{tn})^2 - \frac{k_p}{k_n} (V_{in} - V_{DD} - V_{tp})^2}$$

CMOS Inverter VTC

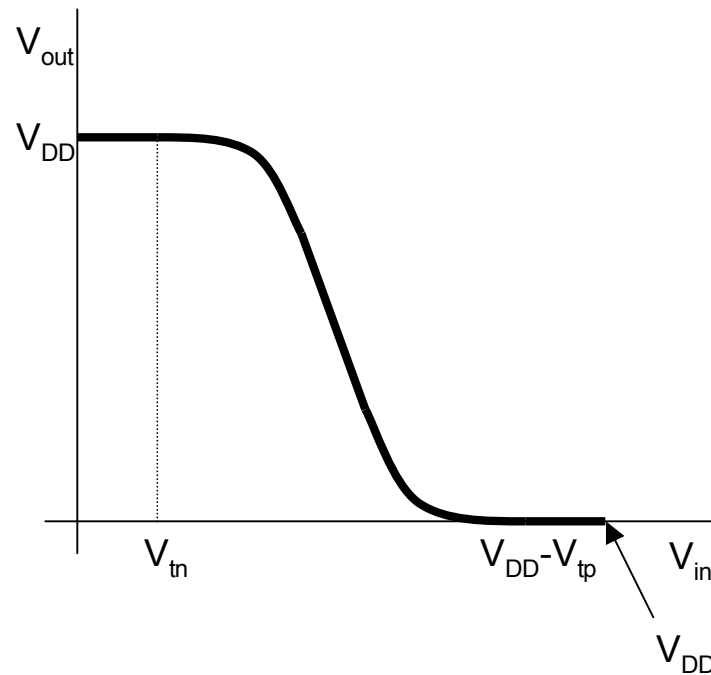
- Short channel model

$$k_p V_{DSATp} \left(V_{in} - V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right) = k_n \left((V_{in} - V_{tn}) V_{out} - \frac{V_{out}^2}{2} \right)$$
$$\frac{V_{out}^2}{2} - (V_{in} - V_{tp}) V_{out} + \frac{k_p}{k_n} V_{DSATp} \left(V_{in} - V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right) = 0$$
$$V_{out} = (V_{in} - V_{tn}) - \sqrt{(V_{in} - V_{tn})^2 - 2 \frac{k_p}{k_n} V_{DSATp} \left(V_{in} - V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right)}$$

CMOS Inverter VTC



CMOS Inverter VTC



CMOS Inverter

V_{in}	pMOS mode	nMOS mode	V_{out}
$V_{in} < V_t$	Linear	Cutoff	V_{DD}
$V_t < V_{in} < V_{out} - V_t$	Linear	Saturation	$(V_{in} + V_t) + \sqrt{(V_{in} - V_{DD} + V_t)^2 - (V_{in} - V_t)^2}$
$V_{out} - V_t < V_{in} < V_{out} + V_t$	Saturation	Saturation	Interpolate
$V_{out} + V_t < V_{in} < V_{DD} - V_t$	Saturation	Linear	$(V_{in} - V_t) - \sqrt{(V_{in} - V_t)^2 - (V_{in} - V_{DD} + V_t)^2}$
$V_{in} > V_{DD} - V_t$	Cutoff	Linear	0

Switching Threshold

- The point at which the inverter has both transistors in saturation

$$\frac{k_n}{2}(V_M - V_{tn})^2 = -\frac{k_p}{2}(V_M - V_{DD} - V_{tp})^2$$

$$(V_M - V_{tn}) = -\sqrt{\frac{-k_p}{k_n}}(V_M - V_{DD} - V_{tp})$$

$$V_M(1 + r) = V_{tn} + r(V_{DD} + V_{tp})$$

$$V_M = \frac{V_{tn} + r(V_{DD} + V_{tp})}{1 + r}$$

Switching Threshold

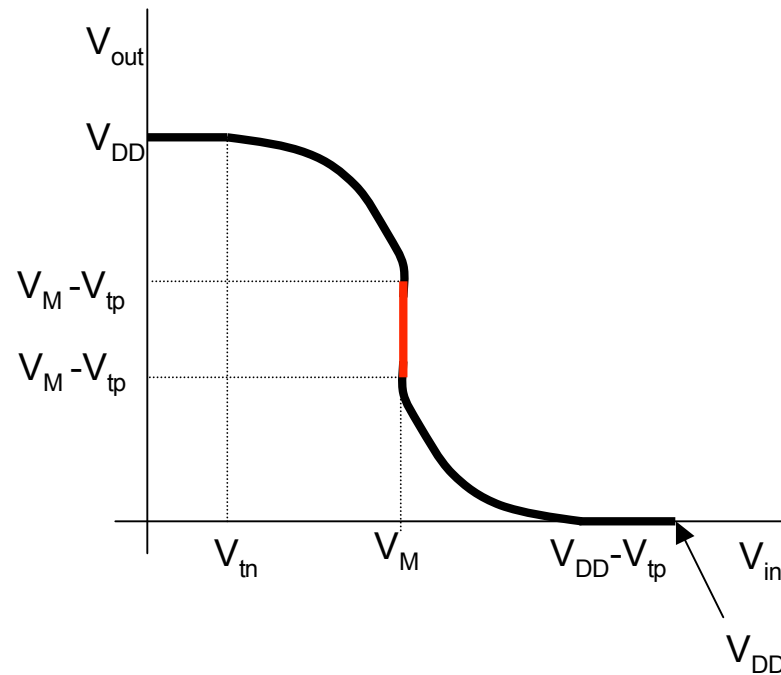
$$V_M = \frac{r(V_{DD} + V_{tp}) + V_{tn}}{1 + r}$$

- When $V_{tn} = -V_{tp}$ and $r=1$,

$$V_M = \frac{V_{DD}}{2}$$

- In switching region, the curve is actually vertical; V_{out} can have multiple values

CMOS Inverter VTC



Switching Threshold

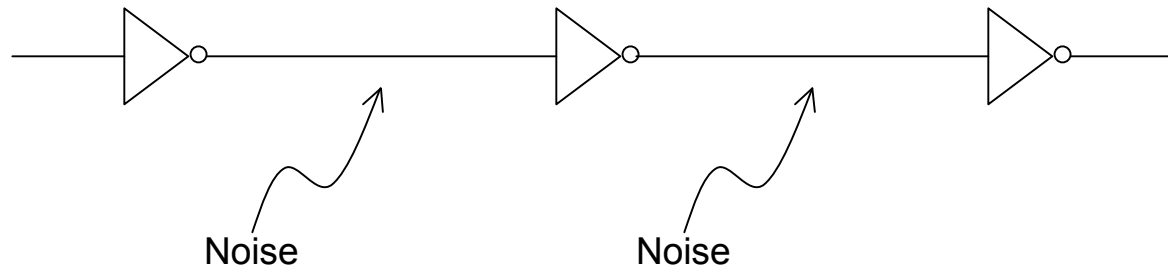
- With short-channel devices

$$\begin{aligned}k_n V_{DSATn} \left(V_M - V_{tn} - \frac{V_{DSATn}}{2} \right) &= -k_p V_{DSATp} \left(V_M - V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right) \\ \left(V_M - V_{tn} - \frac{V_{DSATn}}{2} \right) &= -\frac{k_p V_{DSATp}}{k_n V_{DSATn}} \left(V_M - V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right) \\ V_M \left(1 + \frac{k_p V_{DSATp}}{k_n V_{DSATn}} \right) &= V_{tn} + \frac{V_{DSATn}}{2} - \frac{k_p V_{DSATp}}{k_n V_{DSATn}} \left(-V_{DD} - V_{tp} - \frac{V_{DSATp}}{2} \right) \\ V_M &= \frac{V_{tn} + \frac{V_{DSATn}}{2} + r \left(V_{DD} + V_{tp} + \frac{V_{DSATp}}{2} \right)}{1 + r}\end{aligned}$$

$$V_M = V_{DD} \frac{r}{1 + r}$$

Noise Margin

- A measure of the acceptable noise at a gate input so that the output is not affected.



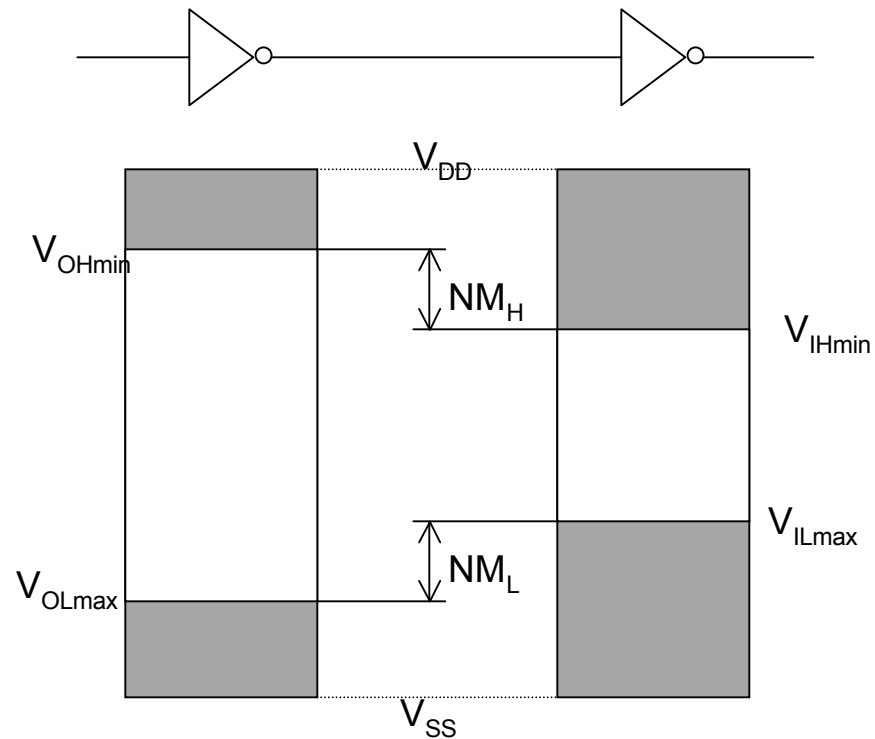
Noise Budget

- Allocates gross noise margin to expected sources of noise
- Sources: supply noise, cross talk, interference, offset
- Differentiate between fixed and proportional noise sources

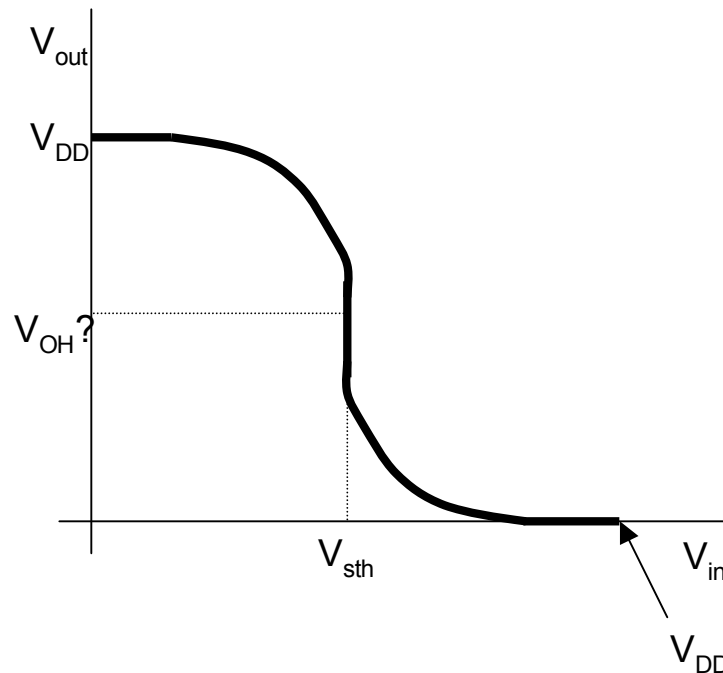
Key Reliability Properties

- Absolute noise margin values are deceptive
 - a floating node is more easily disturbed than a node driven by a low impedance (in terms of voltage)
- Noise immunity is the more important metric – **the capability to suppress noise sources**
- Key metrics: Noise transfer functions, Output impedance of the driver and input impedance of the receiver;

Noise Margins

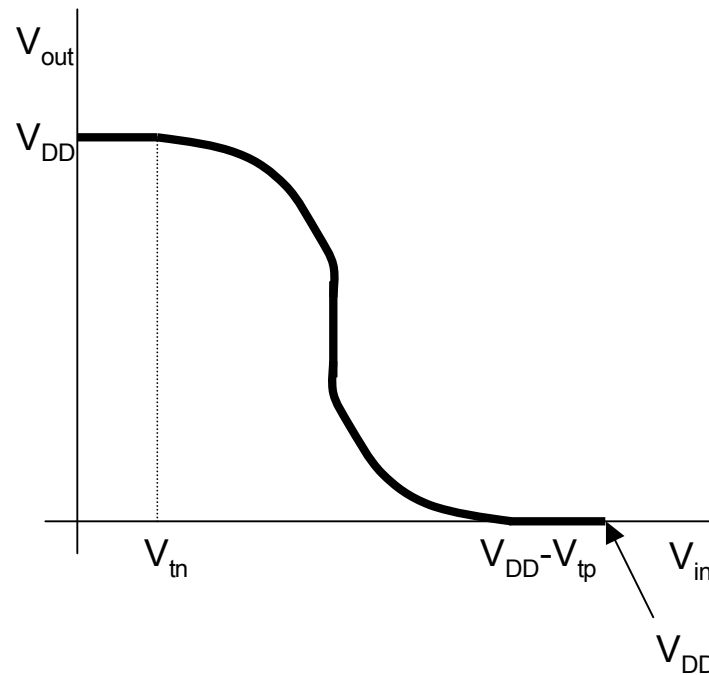


Noise Margins



- Obvious choice
- Set $V_{IH} = V_{IL} = V_M$
- No noise margin
- High gain at V_{IH} and V_{IL}
- Any perturbations could cause incorrect values

Noise Margins



- Set $V_{IL} = V_{tn}$ and $V_{IH} = V_{DD} - V_{tp}$
- $V_{OL} = 0$ and $V_{OH} = V_{DD}$
- Too restrictive

Noise Margins

- Voltage Transfer Function

$$V_{out} = f(V_{in})$$

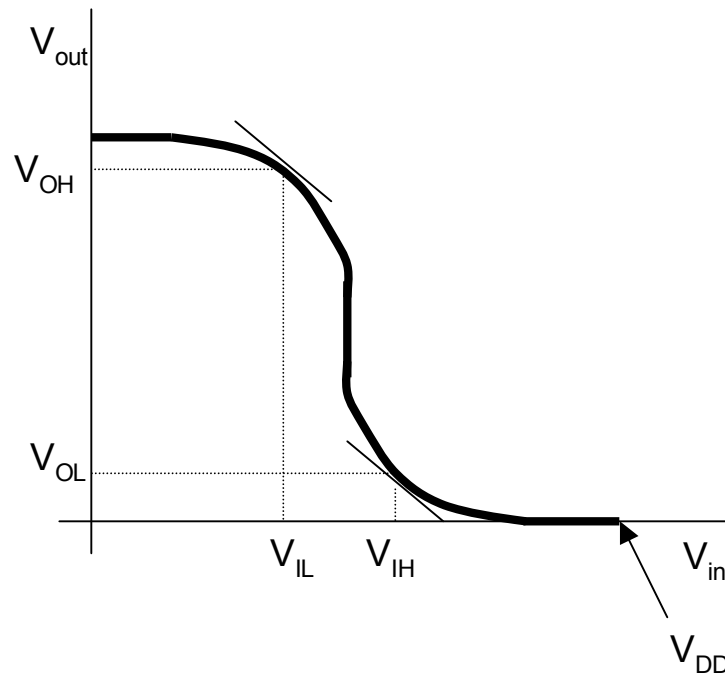
- Voltage Transfer Function with Noise

$$V_{out} = f(V_{in} + \Delta V_{noise})$$

$$V_{out} \approx f(V_{in}) + \frac{dV_{out}}{dV_{in}} \Delta V_{noise}$$

- Perturbed voltage is the sum of the nominal output plus the gain times the noise
- Keep the gain less than 1

Noise Margins



Noise Margins

- Setting the derivative to -1 and solving

$$V_{IH} = V_M + V_M \frac{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)}{1 + r}$$

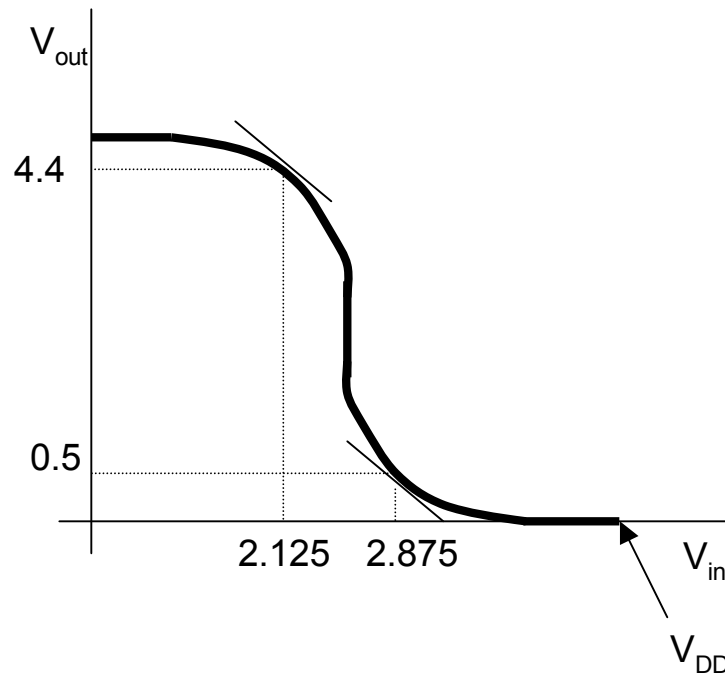
$$V_{IL} = V_M - (V_{DD} - V_M) \frac{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)}{1 + r}$$

- Assuming $r = 1, V_{Tn} = -V_{Tp}, V_{DSATn} = -V_{DSATp}$

$$V_{IH} = \frac{V_{DD}}{2} \left[1 + \left(\frac{V_{DD}}{2} - V_T - \frac{V_{DSAT}}{2} \right) \frac{(\lambda_n - \lambda_p)}{2} \right]$$

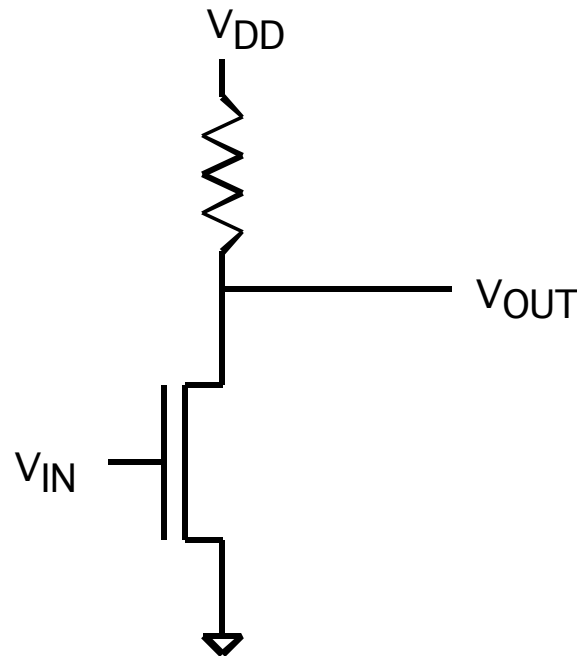
$$V_{IL} = \frac{V_{DD}}{2} \left[1 - \left(\frac{V_{DD}}{2} - V_T - \frac{V_{DSAT}}{2} \right) \frac{(\lambda_n - \lambda_p)}{2} \right]$$

Noise Margins

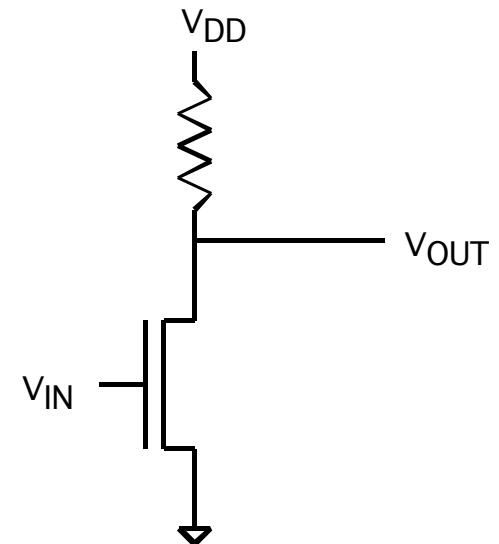
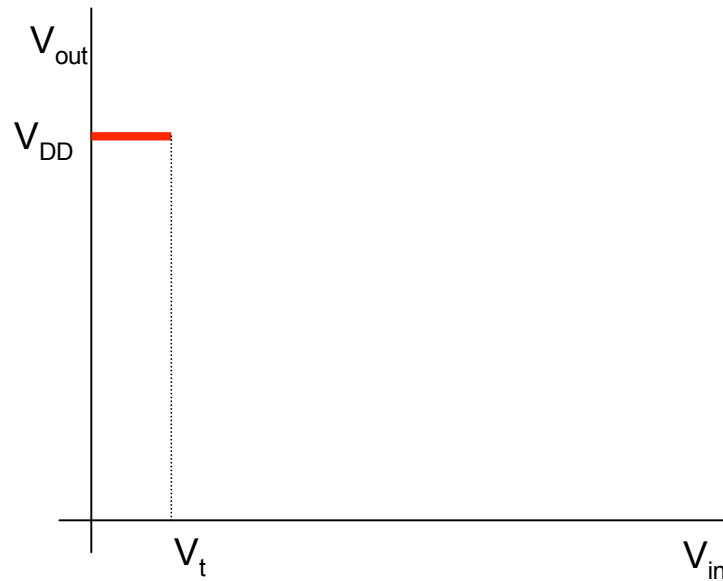


- $V_{DD} = 5\text{ V}$
- $V_t = 0.5\text{ V}$
- $V_{DSAT} = 1.0\text{ V}$
- $\lambda = 0.1$

Static Load Inverter



Static Load Inverter



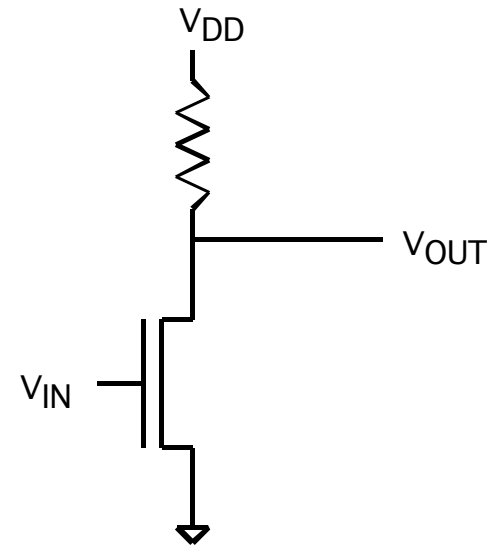
Static Load Inverter

- Transistor is in saturation

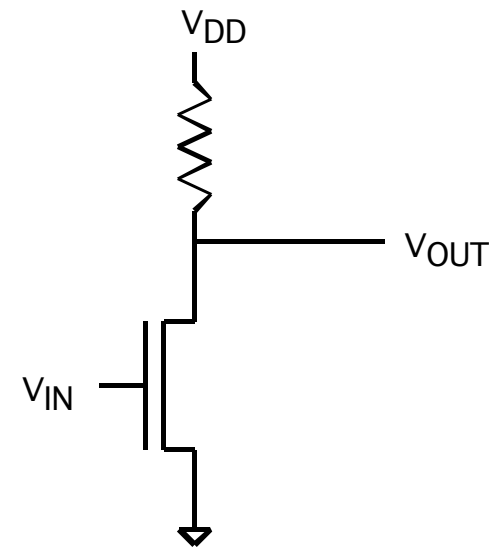
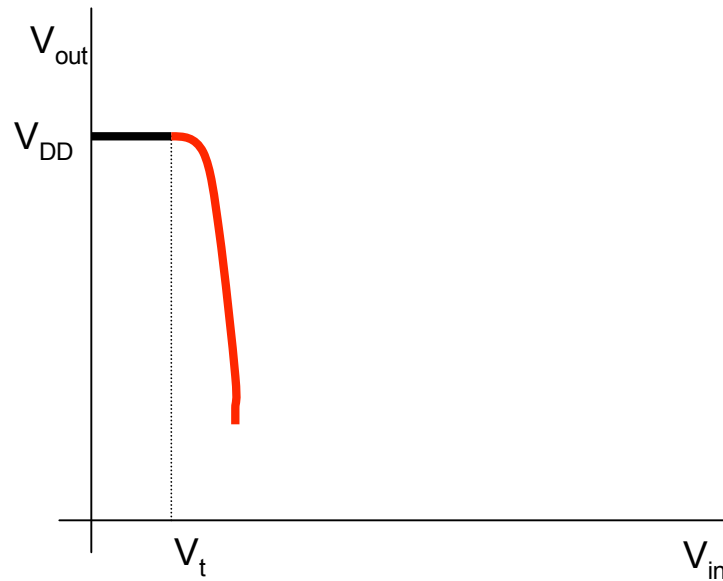
$$I_{DS} = k_n \frac{(V_{GS} - V_T)^2}{2} (1 + \lambda V_{DS})$$

$$V_{OUT} = V_{DD} - Rk_n \frac{(V_{IN} - V_T)^2}{2} (1 + \lambda V_{OUT})$$

$$V_{OUT} = \frac{2V_{DD} - Rk_n (V_{IN} - V_T)^2}{2 + \lambda Rk_n (V_{IN} - V_T)^2}$$



Static Load Inverter



Static Load Inverter

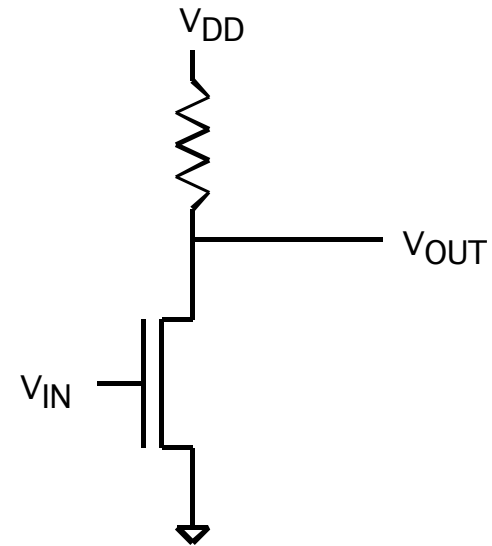
- Transistor is in linear region

$$I_{DS} = k_n \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

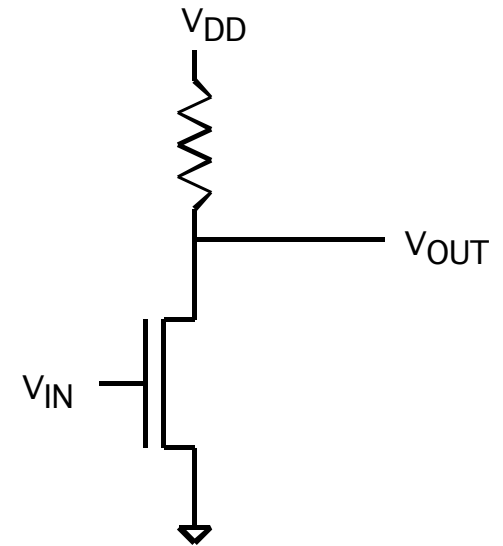
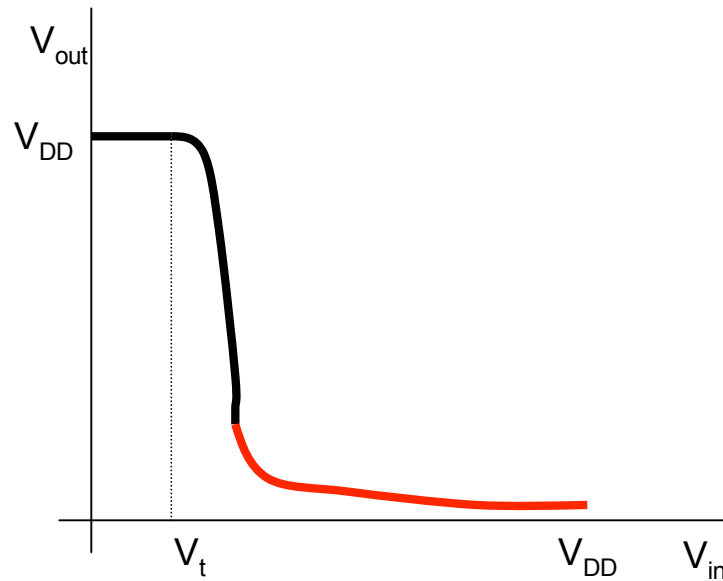
$$V_{OUT} = V_{DD} - Rk_n \left[(V_{IN} - V_T) V_{OUT} - \frac{V_{OUT}^2}{2} \right]$$

$$\frac{Rk_n}{2} V_{OUT}^2 - (Rk_n (V_{IN} - V_T) + 1) V_{OUT} + V_{DD} = 0$$

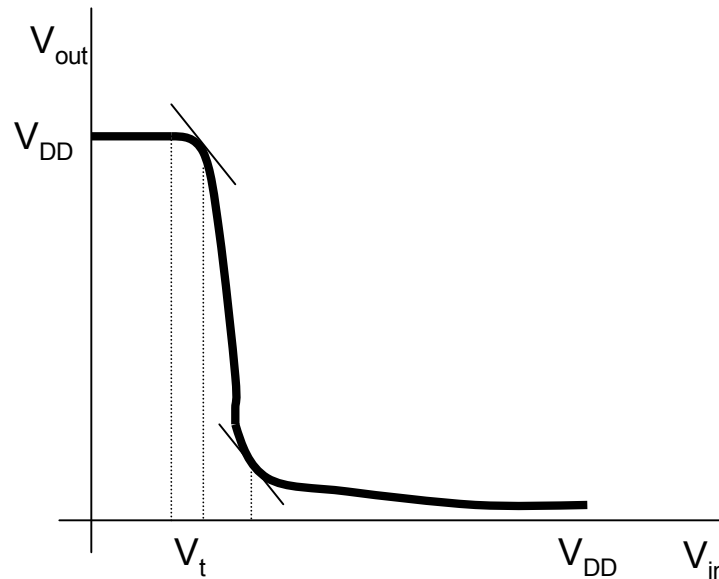
$$V_{OUT} = (V_{IN} - V_T) + \frac{1}{Rk_n} - \sqrt{\left((V_{IN} - V_T) + \frac{1}{Rk_n} \right)^2 - 2 \frac{V_{DD}}{Rk_n}}$$



Static Load Inverter



Static Load Inverter



•Noise Margin

$$V_{IL} = V_T + \frac{1}{k_n R}$$

$$V_{IH} = V_T + \sqrt{\frac{8 V_{DD}}{3 k_n R}} - \frac{1}{k_n R}$$

Static Load Inverter

- Does not go down all the way to 0
- Noise margins are tighter
- Switching threshold is not centered
- To get high gain in the transition region, you need bigger resistors

Homework

- Due February 3rd
- Read Chapter 2